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6.055J / 2.038J The Art of Approximation in Science and Engineering
Spring 2008

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Homework 7

The following problems are for your enjoyment. Happy estimating!

Easier

1. Perfume

If the diffusion constant (in air) for small perfume molecules is $10^{-6} \text{ m}^2 \text{ s}^{-1}$, estimate the time for perfume molecules to diffuse across a room.

Now try the experiment: How long does it take to smell the perfume from across the room? Explain the large discrepancy between the theoretical estimate and the experimental value.

2. Semitones

Estimate 1.5^{14} using semitones and compare with the exact value.

3. Blackbody temperature of the earth

The earth's surface temperature is mostly due to solar radiation.

The solar flux $S \approx 1350 \text{ W m}^{-2}$ is the amount of solar energy reaching the top of the earth's atmosphere. But that energy is spread over the surface of a sphere, so $S/4$ is the relevant flux for calculating the surface temperature. Some of that energy is reflected back to space by clouds or ocean before it can heat the ground, so the heating flux is slightly lower than $S/4$. A useful estimate is $S' \sim 250 \text{ W m}^{-2}$.

Look up the Stefan–Boltzmann law (or see [Problem 7](#)) and use it to find the blackbody temperature of the earth.

Your value should be close to room temperature but enough colder to make you wonder about the discrepancy. Why is the actual average surface temperature warmer than the value calculated in this problem?

4. Xylophone notes

If you double the width, thickness, and length of a xylophone slat, what do you do to the frequency of the note that it makes?

Medium

5. Bending of light

In lecture we estimated how much gravity bends light by using dimensional analysis and then guessing the final functional form. In this problem, you analyze the same situation using discretization.

As before, let r be the closest that the light gets to the origin (which is the center of the gravitating mass). Discretize: Pretend that gravity forgets to deflect the photon unless the photon's distance to the origin is comparable to r . Using that idea, estimate the radial (inward) velocity imparted to the photon, and then the bending angle.

Feel free to neglect dimensionless constants like 2 or π , and check your answer against what we derived in lecture.

6. Rolling down the plane

Four objects, made of identical steel, roll down an inclined plane without slipping. The objects are:

1. a large spherical shell,
2. a large disc,
3. a small solid sphere,
4. a small ring.

The large objects have three times the radius of the small objects. Rank the objects by their acceleration (highest acceleration first).

Check your results with exact calculation or with a home experiment.

7. Blackbody radiation

A hot object – a so-called blackbody – radiates energy, and the flux F depends on the temperature T . In this problem you derive the connection using dimensional analysis. The goal is to find F as a function of T . But you need more quantities.

- a. What are the dimensions of flux?
- b. What two constants of nature should be included because blackbody radiation depends on the quantum theory of radiation?
- c. What constant of nature should be included because you are dealing with temperature?
- d. After doing the preceding parts, you have five variables. Explain why these five variables produce one dimensionless group, and use that fact to deduce the relation between flux and temperature.
- e. Look up the Stefan–Boltzman law and compare your result to it.

Harder

8. Teacup spindown

You stir your afternoon tea to mix the milk (and sugar if you have a sweet tooth). Once you remove the stirring spoon, the rotation starts to slow. What is the spindown time τ ? In other words, how long before the angular velocity of the tea has fallen by a significant fraction?

To estimate τ , consider a physicist's idea of a teacup: a cylinder with height L and diameter L , filled with liquid. Why does the rotation slow? Tea near the edge of the teacup – and near the base, but for simplicity neglect the effect of the base – is slowed by the presence of the edge (the no-slip boundary condition). The edge produces a velocity gradient.

Because of the tea's viscosity, the velocity gradient produces a force on any piece of the edge. This force tries to spin the piece in the direction of the tea's motion. The piece exerts a force on the tea equal in magnitude and opposite in direction. Therefore, the edge slows the rotation. Now you can analyze this model quantitatively.

- a. In terms of the total viscous force F and of the initial angular velocity ω , estimate the spin-down time. Hint: Consider torque and angular momentum. (Feel free to drop any constants, such as π and 2, by invoking the Estimation Theorem: $1 = 2$.)
- b. You can estimate F with the idea that

$$\text{viscous force} \sim \rho v \times \text{velocity gradient} \times \text{surface area.}$$

Here ρv is η . The more familiar viscosity is η , known as the dynamic viscosity. The more convenient viscosity is ν , the kinematic viscosity. The velocity gradient is determined by the size of the region in which the the edge has a significant effect on the flow; this region is called the boundary layer. Let δ be its thickness. Estimate the velocity gradient near the edge in terms of δ , and use the equation for viscous force to estimate F .

- c. Put your expression for F into your earlier estimate for τ , which should now contain only one quantity that you have not yet estimated (the boundary-layer thickness).
- d. You can estimate δ using your knowledge of random walks. The boundary layer is a result of momentum diffusion; just as D is the molecular-diffusion coefficient, ν is the momentum-diffusion coefficient. In a time t , how far can momentum diffuse? This distance is δ . What is a natural estimate for t ? (Hint: After rotating 1 radian, the fluid is moving in a significantly different direction than before, so the momentum fluxes no longer add.) Use that time to estimate δ .
- e. Now put it all together: What is the characteristic spindown time τ (the time for the rotation to slow down by a significant amount)?
- f. Stir some tea to experimentally estimate τ_{exp} . Compare this time with the time predicted by the preceding theory. [In water (and tea is roughly water), $\nu \sim 210^{-6} \text{ m}^2 \text{ s}^{-1}$.]

9. Stokes' law

You can use ideas from the previous problem to derive Stokes' formula for drag at low speeds (more precisely, at low Reynolds' number). Many weeks ago, we derived the result from dimensional analysis; here you will find a physical argument.

Consider a sphere of radius R moving with velocity v . Equivalently, in the reference frame of the sphere, the sphere is fixed and the fluid moves past it with velocity v . Next to the sphere, the fluid is stationary. Over a region of thickness δ (the boundary layer), the fluid velocity rises from zero to the full flow speed v . Assume that $\delta \sim R$ (the most natural assumption) and estimate the viscous drag force. Compare the force with Stokes' formula (remember that $\rho v = \eta$).

10. Bouncing ball

You drop a steel ball, say $r \sim 1 \text{ cm}$, from a height of one or two metres. It lands on a steel surface and bounces up to nearly the original drop height. Estimate the contact force at the instant when the ball is stationary on the ground. Give your answer as the dimensionless ratio

$$\frac{\text{contact force}}{\text{weight of the ball}}$$

Useful data: The elastic modulus of steel is $Y \sim 10^{11} \text{ N m}^{-2}$.