PROFESSOR: There's other kinds of bogus proofs that come up. Let's just run through this one quickly. Here's a fact that you about, roots of polynomials. Every polynomial has two roots, at least over the complex numbers, over c.

And how do you prove that? Well, you just write down the formulas for the roots. You know the quadratic formula. One root is a minus b plus this square root of this quantity of 2a. And the other root is minus b minus this square root of b squared minus 4ac over 2a. And that's the end for the proof.

You can just plug-in this formula for r1 for x into this polynomial and it would simplify to be equal to 0, which shows that this is a root. You could plug that one into this formula for x and simplify algebraically and discover it was 0, proving that r2 is a root. We've just proved that every polynomial has two roots.

Well, that's not true. We haven't proved it. This is a proof by calculation that has problems. What's the problem? well let's look at a counter example. What about the polynomial 0x squared plus 0x plus 1? It doesn't have any roots. It's just a constant 1 which never crosses the origin. So it's got no roots.

What about 0x squared plus 1x plus 1? Well that's 45 degree line, the y equals x line, and it only crosses the origin once. It has only one root. What happened to the two formulas, r1 and r2? And the answer was, in this case, we had to divide by 0 error.

If you look at that formula, there's a quotient, there's a denominator of 2a. Divide by 0 and these formulas don't work right. They aren't the roots. And so implicitly, in order to have two roots, we need to assume that the denominator, a, the leading coefficient of the polynomial is not 0. So that fixes those two bugs.

Is that all? Well, no, because look at this polynomial. 1x squared plus 0x plus 0 has one root. The only possible root of this is 0. Because if you look at this, the only way to get 1 times something plus 0 to equals 0 is if the something is 0. So there's only one root.

And what's going on here? Well, what's happened is that in this case, the two formulas, r1 and r2, which were different formulas, evaluate to the same thing when b is 0 and c is 0 and a is 1. And that's why they look like different formulas but they evaluate to the same thing so there's only one root.

The fix to that is to require the quantity by which the two root formulas, r1 and r2 differ to be non-zero. And that's the quantity that we were taking the square root of, the discriminant it's called. b squared minus 4ac needs to be 0 and then r1 and r2 will differ and we will get the two roots.

Now, there's still a complication. It sounds like we've now verified that indeed our proof by calculation is correct now that we've put in these qualifiers, that a is positive and d is non-zero. But what happens if d is non-zero but negative? It's an interesting complication. And let's look at the formula, x squared plus 1, where b squared minus 4ac is going to be minus 3. And that will turn out to have two roots, namely i and minus i.

And it's not possible to tell which is which. r1 is taking the square root of minus 1, and r2 is taking the square root of minus 1. One of them is adding the square root of minus 1. The other one's subtracting the square root of minus 1. But which square root of minus 1? There's no way to tell the difference between i and minus i, abstractly. They both behave the same way. And so we have an ambiguity about which one is r1 and which one is r2. It doesn't hurt at all for the theorem that r1 and r2 are different. And so there are two roots.

But ambiguity can be problematic. And let me give you an illustration of that. When there's ambiguity, I can do things like proving easily that 1 is equal to minus 1. Here's the proof. And I will let you contemplate that and try to figure out just where in this reasoning that step by step seems pretty reasonable, but nevertheless I've concluded that 1 is equal to minus 1, which is absurd. It's taking advantage of the fact that you don't know whether the square root of minus 1 means i or minus i.

So the moral of all of this is that, first of all, be sure that you are applying the rules properly. There's an assumption of an algebraic rule in there that isn't true. And again, that kind of mindless calculation is risky when you don't really understand what you're doing, you don't have a clear memory of what the exact rules are. So it's understanding that bails you out of this kind of blunder.

Let's look at 1 equals minus 1 a little because it lets us wrap up with an amusing remark. What's terrible about 1 equals minus 1? Well, it's false, and you don't want to ever conclude something that's false. That's worrisome. It's disastrous when you conclude that something is false.

Let me give you an illustration of why. Because let's suppose the 1 is equal to minus 1 and let's reason in a correct from that assumption that we have falsely proved. But let's assume that we start off with the assumption that 1 is minus 1. Well, if I multiply both sides of an equation by the same thing, it's equal.

So I can multiply both sides by 1/2, and I get 1/2 is equal to minus 1/2. Now I can also add the same thing to both sides. That's a perfectly sound move for reasoning about equalities. If I add 3/2 to both sides, I've turn 1 equals minus 1 into 2 is equal to 1.

Now I'm in great shape to prove all kinds of things. Here's a famous one. "Since I am the Pope are clearly 2, we conclude that I and the Pope are one. That is, I am the Pope." And I've just proved to you this absurd fact.

This is a joke that's attributed-- a witty remark that's attributed to Bertrand Russell, the famous philosopher, logician, pacifist, Nobel Prize winner, who supposedly was approached by some socialite at a party who had heard that mathematicians thought that if 1 is equal to minus 1 then you could prove anything. And so she challenged him, prove that you're the Pope. And supposedly Russell, who was a notoriously quick wit, came up with this example. Who knows whether it's true. It's a good story.

There's a picture of the great man. And you might care to learn more about this remarkable contributor to logic, and philosophy, and politics.