Mathematics for Computer Science
MIT 6.042J/18.062J

The Well Ordering Principle, II

Prime Products

Thm: Every integer > 1 is a product of primes.

Proof: (by contradiction) Suppose {nonproducts} is nonempty. By WOP, there is a least $m > 1$ that is a nonproduct. This m is not prime (else is a product of 1 prime)

Prime Products

Thm: Every integer > 1 is a product of primes.

...So $m = j \cdot k$ for integers j,k where $m > j,k > 1$. Now $j,k < m$ so both are prime products:

$j = p_1 \cdot p_2 \cdots p_{94}$ $k = q_1 \cdot q_2 \cdots q_{213}$

Prime Products

Thm: Every integer > 1 is a product of primes.

...now

$m = j \cdot k = p_1 \cdot p_2 \cdots p_{94} \cdot q_1 \cdot q_2 \cdots q_{213}$

is prime product, contradiction.

So {counterexamples} = $\varnothing$. QED

Well Ordered Postage

available stamps:


5¢ 3¢

n is postal if can make (n+8)¢ postage from 3¢ & 5¢ stamps.

Well Ordered Postage

available stamps:


5¢ 3¢

Thm: Every number is postal.

Prove by WOP. Suppose not. Let m be least counterexample.

Well Ordered Postage

available stamps: 

5¢ 3¢

That is,

- m is not postal,
- any number < m is postal

Well Ordered Postage

0 is postal: 

so m ≠ 0

Well Ordered Postage

m ≠ 1: 

m ≠ 2: 

Hence, m ≥ 3.

Well Ordered Postage

Now m-3 is a number < m, so is postal. But then m is postal too:

(m-3)+8¢ + 3¢ = m+8¢

contradiction!

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