

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

*Mathematics for Computer Science*  
MIT 6.042J/18.062J

## The Well Ordering Principle, II

This work is licensed under a [Creative Commons Attribution-NonCommercial-Share Alike 3.0 Unported License](https://creativecommons.org/licenses/by-nc-sa/4.0/).

Albert R Meyer      February 13, 2012      Lec 2M.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Prime Products

*Thm:* Every integer  $> 1$  is a product of primes.

*Proof:* (by contradiction) Suppose {nonproducts} is nonempty. By WOP, there is a **least**  $m > 1$  that is a nonproduct. This  $m$  is not prime (else is a product of 1 prime)

Albert R Meyer      February 13, 2012      Lec 2M.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Prime Products

*Thm:* Every integer  $> 1$  is a product of primes.

...So  $m = j \cdot k$  for integers  $j, k$  where  $m > j, k > 1$ . Now  $j, k < m$  so both are prime products:  
 $j = p_1 \cdot p_2 \cdots p_{94}$      $k = q_1 \cdot q_2 \cdots q_{213}$

Albert R Meyer      February 13, 2012      Lec 2M.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Prime Products

*Thm:* Every integer  $> 1$  is a product of primes.

...now  
 $m = j \cdot k = p_1 \cdot p_2 \cdots p_{94} \cdot q_1 \cdot q_2 \cdots q_{213}$   
 is prime product, **contradiction**.  
 So {counterexamples} =  $\emptyset$ . **QED**

Albert R Meyer      February 13, 2012      Lec 2M.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Well Ordered Postage

available stamps:  

$5\text{¢}$        $3\text{¢}$

$n$  is **postal** if can make  $(n+8)\text{¢}$  postage from  $3\text{¢}$  &  $5\text{¢}$  stamps.

Albert R Meyer      February 13, 2012      Lec 2M.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Well Ordered Postage

available stamps:  

$5\text{¢}$        $3\text{¢}$

*Thm:* Every number is **postal**.  
 Prove by WOP. Suppose **not**.  
 Let  $m$  be **least** counterexample.

Albert R Meyer      February 13, 2012      Lec 2M.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

### Well Ordered Postage

available stamps:  

That is, 5¢ 3¢

- $m$  is not postal,
- any number  $< m$  is postal

 Albert R Meyer February 13, 2012 Lec 2M.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

### Well Ordered Postage

0 is postal:  

so  $m \neq 0$

 Albert R Meyer February 13, 2012 Lec 2M.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

### Well Ordered Postage

$m \neq 1$ : 

$m \neq 2$ : 

Hence,  $m \geq 3$ .

 Albert R Meyer February 13, 2012 Lec 2M.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

### Well Ordered Postage

Now  $m-3$  is a number  $< m$ , so is postal. But then  $m$  is postal too:

 +  =  $m+8\text{¢}$   
3¢

$(m-3)+8\text{¢}$  contradiction!

 Albert R Meyer February 13, 2012 Lec 2M.10

MIT OpenCourseWare  
<http://ocw.mit.edu>

6.042J / 18.062J Mathematics for Computer Science  
Spring 2015

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.