

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

The Well Ordering Principle

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Albert R Meyer February 13, 2012 Lec 2M.1

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Well Ordering principle

Every nonempty set of *nonnegative integers* has a *least element*.

Familiar? Now you mention it, *Yes*.
Obvious? *Yes*.
Trivial? *Yes*. But *watch out*!

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6	9	13	7
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Well Ordering principle

Every nonempty set of *nonnegative rationals* has a *least element*.

NO!

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Well Ordering principle

Every nonempty set of ~~*nonnegative integers*~~ has a *least element*.

NO!

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What is the

- *youngest* age of MIT graduate?
- *smallest* # neurons in any animal?
- *smallest* #coins = \$1.17?

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$\mathbb{N} ::=$ nonnegative integers

For rest of this talk, "number" means nonnegative integer

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$\sqrt{2}$ proof used Well Ordering

Proof: ...suppose $\sqrt{2} = \frac{m}{n}$

...can **always** find such $m, n > 0$
without common factors...

why **always**?



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Proof using Well Ordering

Find **smallest** number m s.t.

$\sqrt{2} = \frac{m}{n}$. If m, n had a
common factor, $c > 1$, then

$\sqrt{2} = \frac{(m/c)}{(n/c)}$ and $m/c < m$



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Proof using Well Ordering

Find **smallest** number m s.t.

$\sqrt{2} = \frac{m}{n}$.

This **contradiction** implies
 m, n have no common factors.



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