

6	13	7
12	10	5
3	1	4
15	8	11
2	9	14

Mathematics for Computer Science
MIT 6.042J/18.062J

The Pigeonhole Principle



Albert R Meyer, April 24, 2013

pigeonhole.1

6	13	7
12	10	5
3	1	4
15	8	11
2	9	14

Pigeonhole Principle

If **more** pigeons



than pigeonholes,



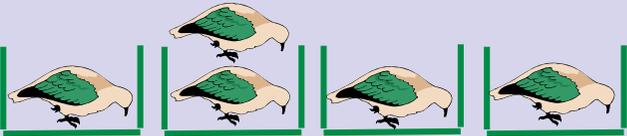

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pigeonhole.2

6	13	7
12	10	5
3	1	4
15	8	11
2	9	14

Pigeonhole Principle

then **some hole** must
have \geq **two** pigeons!




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pigeonhole.3

6	13	7
12	10	5
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Pigeonhole Principle

Mapping Rule: **total injection** from
A to B implies $|A| \leq |B|$.

If $|A| > |B|$, then
no total injection from A to B.



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pigeonhole.4

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 **example: 5 Card Draw**

set of 5 cards:
must have ≥ 2
with the same suit.

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 **5 Card Draw**

5 cards (pigeons) 

4 suits (holes) 

Diagram showing 5 cards being placed into 4 suit holes. Arrows point from each of the 5 cards to one of the 4 holes.

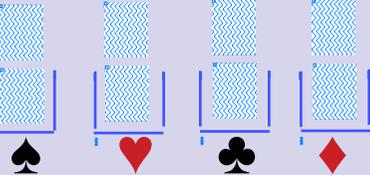
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 **10 Card Draw**

10 cards: how many have
the same suit?

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 **10 Card Draw**

NO!

< 3 cards in every hole?

Diagram showing 5 cards being placed into 4 suit holes. The fifth hole is empty and has a red 'NO!' written next to it. Below the diagram, the text asks if there are fewer than 3 cards in every hole.

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8	13	7
12	10	5
3	1	14
15	6	11

10 Card Draw

cards with same suit

$$\geq \left\lceil \frac{10}{4} \right\rceil = 3$$

"ceiling," means round up



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pigeonhole.10

8	13	7
12	10	5
3	1	14
15	6	11

Generalized Pigeonhole Principle

If n pigeons and h holes,
then some hole has \geq

$$\left\lceil \frac{n}{h} \right\rceil \text{ pigeons.}$$



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pigeonhole.11

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