

Mathematics for Computer Science
MIT 6.042J/18.062J

Rules for Counting



Albert R Meyer, April 17, 2013

rulescount.1

Counting in Gambling

What *fraction* of poker hands
are "a pair of Jacks?"
(*probability* of a pair
of Jacks)



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rulescount.2

Counting in Games

- # different chess positions
after n moves?
- # different positions
for a Rubik's cube?



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rulescount.3

Counting in Algorithms

- # ops to update a data
structure (# comparisons
needed to sort n items)
- # steps in a computation (#
multiplies to compute d^n)



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rulescount.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Counting in **Cryptography**

possible passwords

possible keys

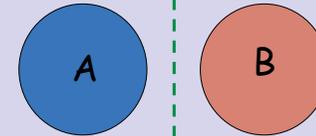


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rulescount.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Sum Rule



If sets A and B are **disjoint**, then

$$|A \cup B| = |A| + |B|$$



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rulescount.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Sum Rule

- Class has 43 women, 54 men so total enrollment = $43 + 54 = 97$
- 26 lower case letters, 26 upper case letters, and 10 digits, so # characters = $26+26+10 = 62$



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rulescount.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Product Rule

If there are 4 boys and 3 girls, there are

$$4 \cdot 3 = 12$$

different boy/girl couples



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rulescount.10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Product Rule

If $|A| = m$ and $|B| = n$, then
 $|A \times B| = m \cdot n$

$A = \{a, b, c, d\}$, $B = \{1, 2, 3\}$

$A \times B = \{(a,1),(a,2),(a,3),$
 $(b,1),(b,2),(b,3),$
 $(c,1),(c,2),(c,3),$
 $(d,1),(d,2),(d,3)\}$



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rulescount.11

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Product Rule: Counting Strings

length-4 binary strings

$= |B \times B \times B \times B|$

$= |B^4|$ where $B ::= \{0,1\}$

$= 2 \cdot 2 \cdot 2 \cdot 2 = 2^4$



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rulescount.12

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Product Rule: Counting Strings

length n strings
 from an alphabet of
 size m is

m^n



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rulescount.13

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