

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science

MIT 6.042J/18.062J

Sets: operations



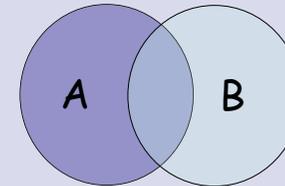
Albert R Meyer

February 19, 2014

sets-ops.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

New sets from old



Venn Diagram for 2 Sets



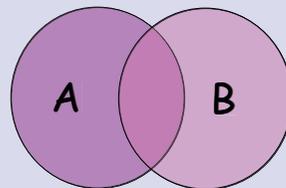
Albert R Meyer

February 19, 2014

sets-ops.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

union



$$A \cup B ::= \{x \mid x \in A \text{ OR } x \in B\}$$



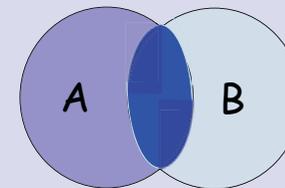
Albert R Meyer

February 19, 2014

sets-ops.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

intersection



$$A \cap B ::= \{x \mid x \in A \text{ AND } x \in B\}$$



Albert R Meyer

February 19, 2014

sets-ops.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

A set-theoretic equality

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

proof: Show these have the same elements, namely,
 $x \in \text{Left Hand Set}$ iff $x \in \text{RHS}$
 for all x .



Albert R Meyer

February 19, 2014

sets-ops.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

A set-theoretic equality

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

proof uses fact from last time:

$$P \text{ OR } (Q \text{ AND } R) \text{ equiv} \\ (P \text{ OR } Q) \text{ AND } (P \text{ OR } R)$$



Albert R Meyer

February 19, 2014

sets-ops.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

A set-theoretic equality

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

proof: $x \in A \cup (B \cap C)$ iff
 $x \in A \text{ OR } x \in (B \cap C)$ (def of \cup) iff
 $x \in A \text{ OR } (x \in B \text{ AND } x \in C)$ (def \cap) iff
 $(x \in A \text{ OR } x \in B) \text{ AND } (x \in A \text{ OR } x \in C)$
 (by the equivalence)



Albert R Meyer

February 19, 2014

sets-ops.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

A set-theoretic equality

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

proof: $x \in A \cup (B \cap C)$ iff
 $x \in A \text{ OR } x \in (B \cap C)$ (def of \cup) iff
 $P \text{ OR } (Q \text{ AND } R)$ (def \cap) iff
 $(P \text{ OR } Q) \text{ AND } (P \text{ OR } R)$
 (by the equivalence)



Albert R Meyer

February 19, 2014

sets-ops.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

A set-theoretic equality

proof:

$(x \in A \text{ OR } x \in B) \text{ AND } (x \in A \text{ OR } x \in C)$ iff
 $(x \in A \cup B) \text{ AND } (x \in A \cup C)$ (def \cup) iff
 $x \in (A \cup B) \cap (A \cup C)$ (def \cap).

QED



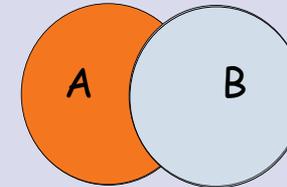
Albert R Meyer

February 19, 2014

sets-ops.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

difference



$$A - B ::= \{x \mid x \in A \text{ AND } x \notin B\}$$



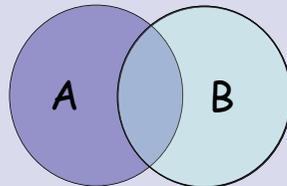
Albert R Meyer

February 19, 2014

sets-ops.10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

complement



$$\bar{A} ::= D - A = \{x \mid x \notin A\}$$



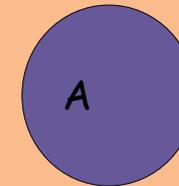
Albert R Meyer

February 19, 2014

sets-ops.12

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

complement



$$\bar{A} ::= D - A = \{x \mid x \notin A\}$$



Albert R Meyer

February 19, 2014

sets-ops.13

MIT OpenCourseWare
<http://ocw.mit.edu>

6.042J / 18.062J Mathematics for Computer Science
Spring 2015

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.