Mathematics for Computer Science

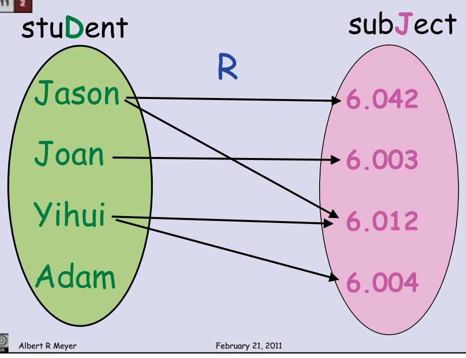
MIT 6.042J/18.062J

Relations & Functions

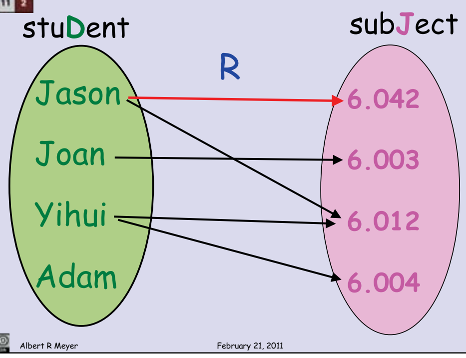
Binary relations

A binary relation associates elements of one set called the domain, with elements of another set called the codomain

"Registered for" relation R



"Registered for" relation R



"Registered for" relation R

Jason is registered for 6.042

notation:

Jason R 6.042 infix

R(Jason, 6.042) prefix

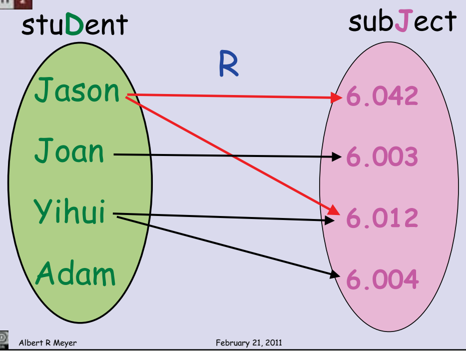
(Jason, 6.042) ∈ R

(Jason, 6.042) ∈ graph(R)

Images under R

R(Jason) = subjects Jason is registered for

"Registered for" relation R



Images under R

R(Jason) = subjects Jason is registered for

= {6.042, 6.012}

Images under R

R(X) ::= all the subjects being taken by students in the set X

Images under R

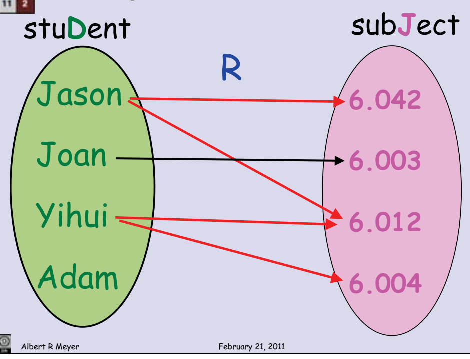
R(X) ::= everything R relates to things in X

Images under R

R({Jason, Yihui}) =

subjects with Jason or Yihui registered

"Registered for" relation R



Images under R

$R(\{Jason, Yihui\})$ =

subjects with Jason or Yihui registered

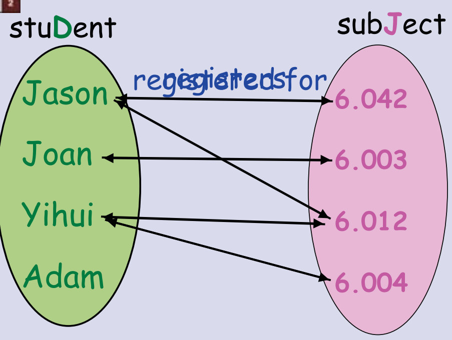
= {6.042, 6.012, 6.004}

Images under R

$R(X)$ ::= endpoints of arrows from points in X

$$\{j \in J \mid \underbrace{\exists d \in X.\ d\,R\,j}_{\text{an arrow from } X \text{ goes to } j}\}$$

"registers" relation R^{-1}



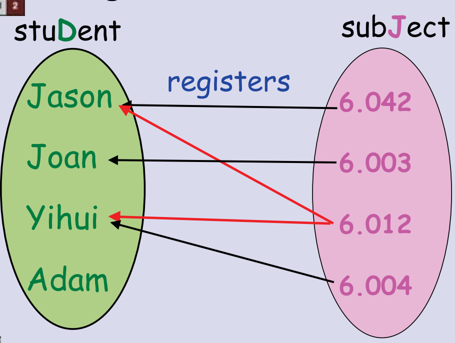
"registers" relation R^{-1}

$$d\,R\,j \quad \text{IFF} \quad j\,R^{-1}\,d$$

Images under R^{-1}

$R^{-1}(6.012)$ =

"registers" relation R^{-1}



Images under R^{-1}

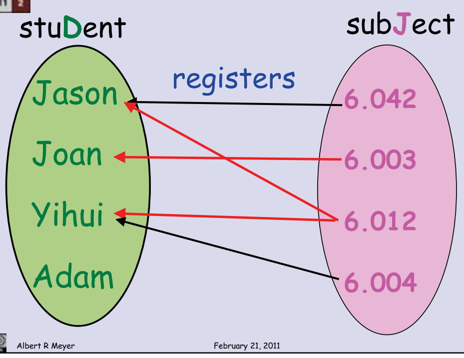
$R^{-1}(6.012) = \{Jason, Yihui\}$

Images under R^{-1}

$R^{-1}(6.012) = \{Jason, Yihui\}$

$R^{-1}(\{6.012, 6.003\}) =$

"registers" relation R^{-1}



Images under R^{-1}

$R^{-1}(6.012) = \{Jason, Yihui\}$

$R^{-1}(\{6.012, 6.003\}) =$

$\{Jason, Joan, Yihui\}$

$R^{-1}(Y)$ aka the inverse image of Y under R

Inverse image under R

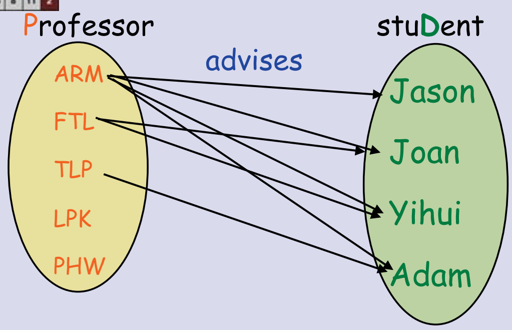
$R^{-1}(J) =$ all the stuDents registered for some subJect

Every student is registered for some subject:

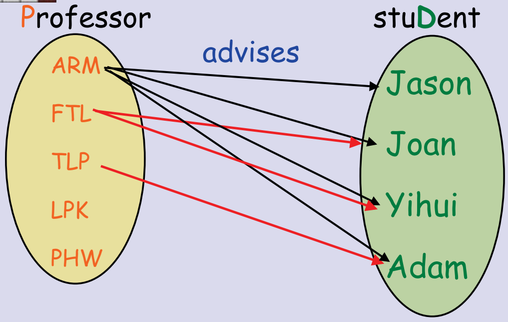
$$D \subseteq R^{-1}(J)$$

(not true: Adam wasn't registered)

"advises" relation V



"advises" relation V



Composing R and V

$R(V(\{FTL, TLP\})) = R(\{Joan, Yihui, Adam\})$

Composing R and V

$R(V(\{FTL, TLP\})) = R(\{Joan, Yihui, Adam\})$

$= \{6.003, 6.012, 6.004\}$

$R(V(X)) =$ subJects that advisees of profs in X are registered for

Composing R and V

$$(R \circ V)(X) ::= R(V(X))$$

$$R \circ V$$

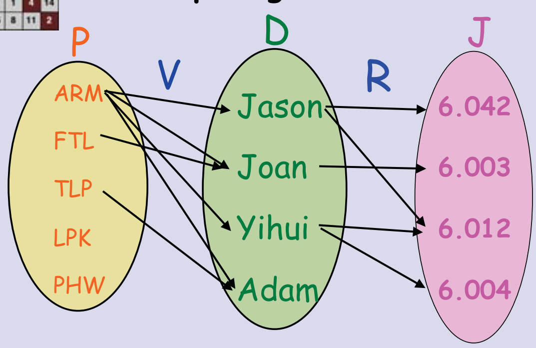
is the composition of R and V

Composing R and V

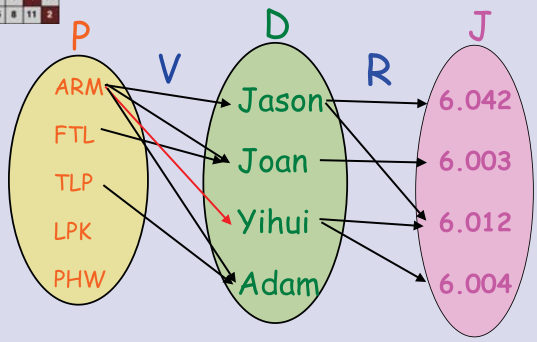
$R \circ V$ ::= "prof has advisee registered for"

$p(R \circ V)j$ ::= prof p has an advisee registered in subject j

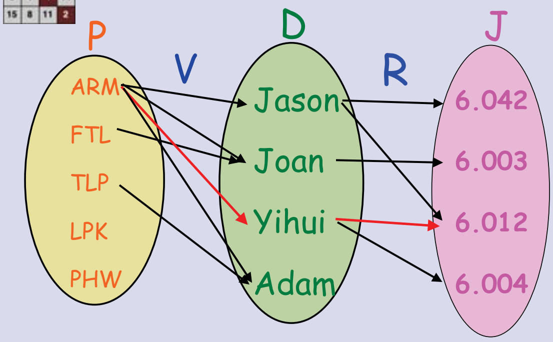
Composing R and V



Composing R and V



Composing R and V



Composing R and V

ARM $(R \circ V)$ 6.012 because

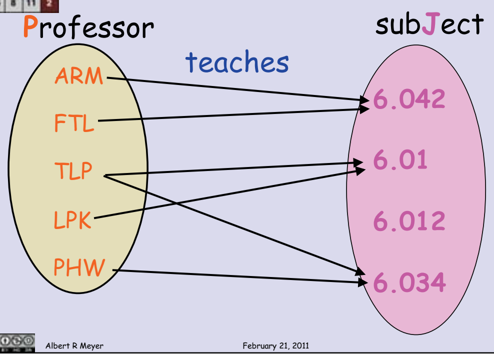
ARM V Yihui AND Yihui R 6.012

$p(\underbrace{R \circ V})j$ IFF

$$\exists d \in D.\ \left[p\ \underbrace{V\ d \ \text{AND}\ d\ R}\ j\right]$$

note: V,R in reverse order

"teaches" relation T



set operations on relations

Profs should not teach their advisees:

$$\forall p\ \forall j.\ \text{NOT}\left(p(R \circ V)j \ \text{ AND }\ pTj\right)$$

$$T \cap (R \circ V) = \varnothing$$

set operations on relations

Profs should not teach their advisees:

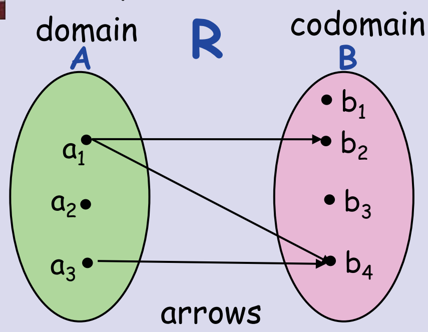
$$\forall p\ \forall j.\ \text{NOT}\left(p(R \circ V)j \ \text{ AND }\ pTj\right)$$

$$R \circ V \subseteq \overline{T}$$

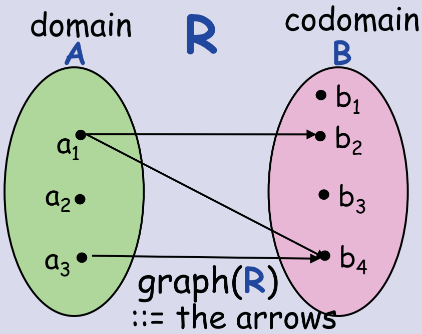
Binary relations

A binary relation, R, from a set A to a set B associates elements of A with elements of B.

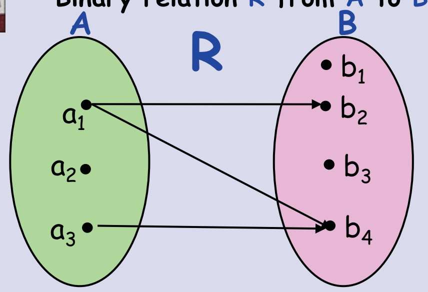
Binary relation R from A to B



Binary relation R from A to B



Binary relation R from A to B



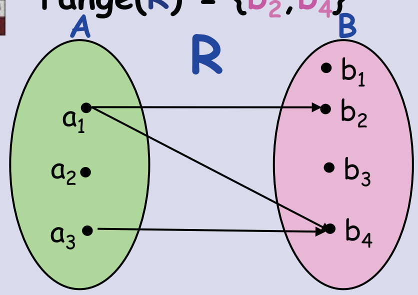
graph(R) = $\{(a_1,b_2), (a_1,b_4), (a_3,b_4)\}$

range(R)

range(R) ::= elements with arrows coming in

= R(A)

range(R) = $\{b_2, b_4\}$

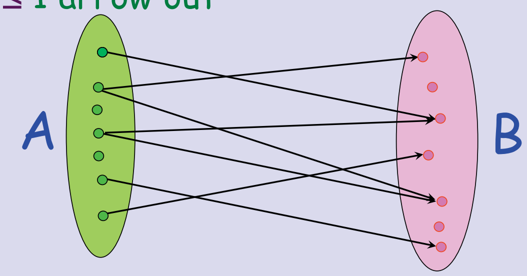


Functions are relations

A function, F, from A to B is a relation which associates each element, a, of A with at most $\underbrace{\text{one element of B}}_{\text{called } F(a)}$.

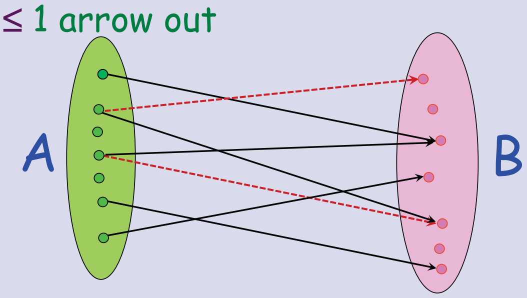
function archery

≤ 1 arrow out



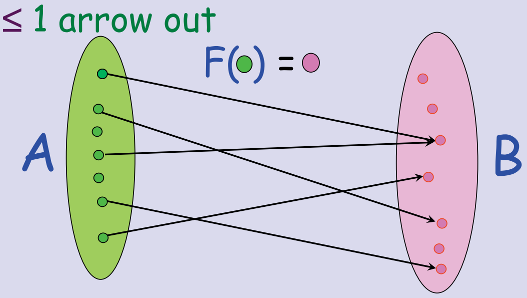
function archery

≤ 1 arrow out



function archery

≤ 1 arrow out



Functions are relations

relation $F{:}A \to B$ is a function

IFF $|F(a)| \le 1$

IFF

$[a\,F\,b \text{ AND } a\,F\,b']$ IMPLIES $b = b'$