

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science
 MIT 6.042J/18.062J

Relations & Functions

Albert R Meyer February 21, 2011 lec 3T.1

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Binary relations

A **binary relation** associates elements of one set called the **domain**, with elements of another set called the **codomain**

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

"Registered for" relation R

stuDent
 Jason
 Joan
 Yihui
 Adam

R

subJect
 6.042
 6.003
 6.012
 6.004

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

"Registered for" relation R

stuDent
 Jason
 Joan
 Yihui
 Adam

R

subJect
 6.042
 6.003
 6.012
 6.004

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

"Registered for" relation R

Jason is registered for 6.042

notation:

Jason R 6.042 infix
 $R(\text{Jason}, 6.042)$ prefix
 $(\text{Jason}, 6.042) \in R$
 $(\text{Jason}, 6.042) \in \text{graph}(R)$

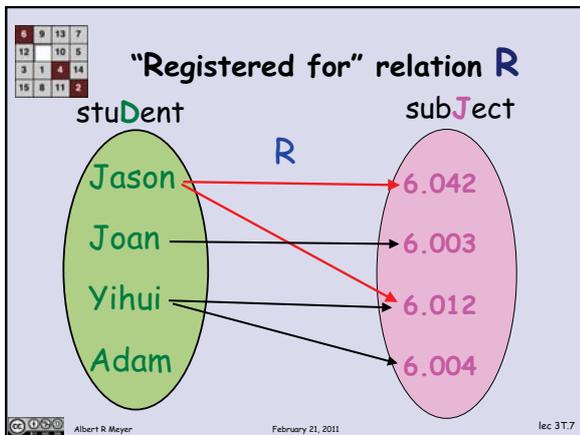
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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Images under R

$R(\text{Jason}) =$ subjects Jason is registered for

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R

$R(\text{Jason}) =$ subjects Jason is registered for
 $= \{6.042, 6.012\}$

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R

$R(X) ::=$ all the subjects being taken by students in the set X

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R

$R(X) ::=$ everything R relates to things in X

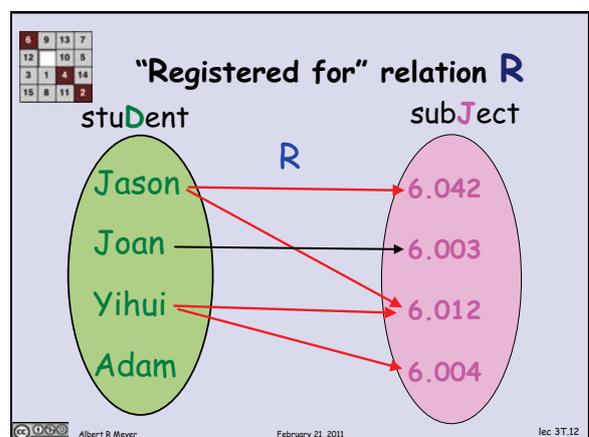
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R

$R(\{\text{Jason}, \text{Yihui}\}) =$
 subjects with Jason or Yihui registered

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R

$R(\{\text{Jason}, \text{Yihui}\}) =$
 subjects with Jason
 or Yihui registered
 $= \{6.042, 6.012, 6.004\}$

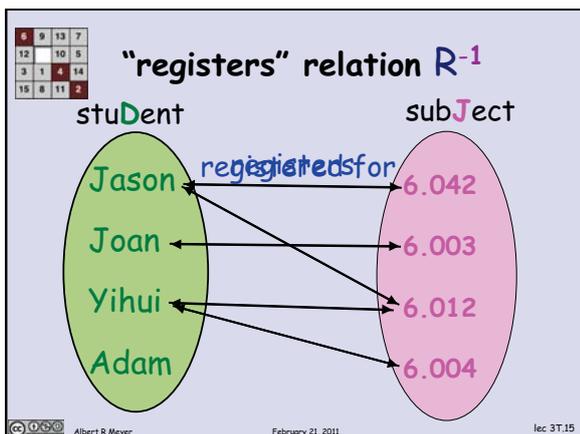
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R

$R(X) ::=$ endpoints of
 arrows from points in X
 $\{j \in J \mid \exists d \in X. d R j\}$
 an arrow from X goes to j

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

"registers" relation R^{-1}

$d R j \quad \text{IFF} \quad j R^{-1} d$

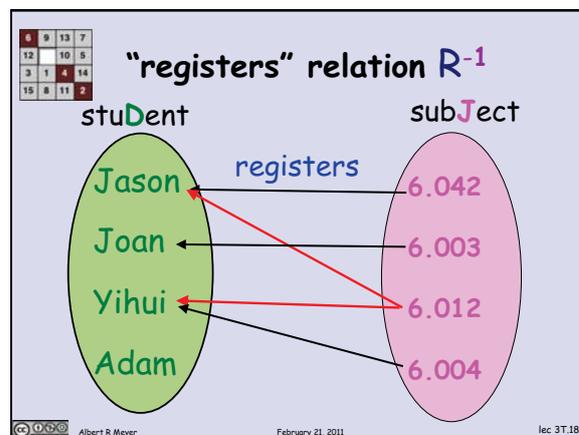
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R^{-1}

$R^{-1}(6.012) =$

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R^{-1}

$R^{-1}(6.012) = \{\text{Jason, Yihui}\}$

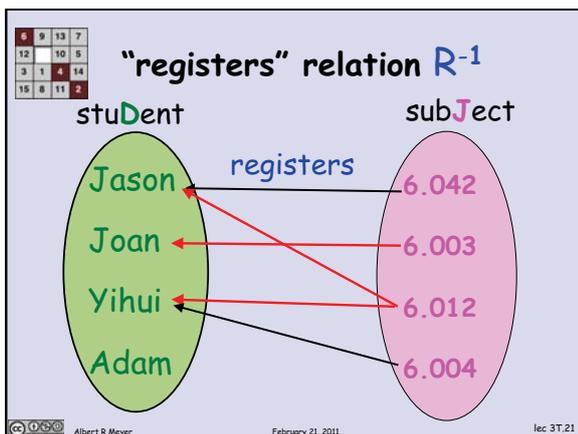
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R^{-1}

$R^{-1}(6.012) = \{\text{Jason, Yihui}\}$
 $R^{-1}(\{6.012, 6.003\}) =$

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6	9	13	7
12		10	5
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15	8	11	2

Images under R^{-1}

$R^{-1}(6.012) = \{\text{Jason, Yihui}\}$
 $R^{-1}(\{6.012, 6.003\}) =$
 $\{\text{Jason, Joan, Yihui}\}$
 $R^{-1}(Y)$ aka the **inverse image**
of Y under R

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6	9	13	7
12		10	5
3	1	4	14
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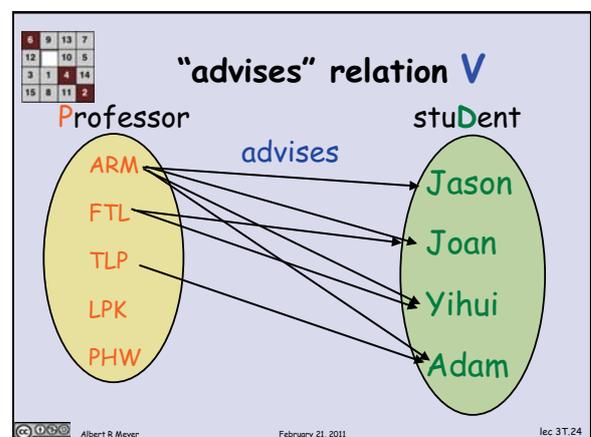
Inverse image under R

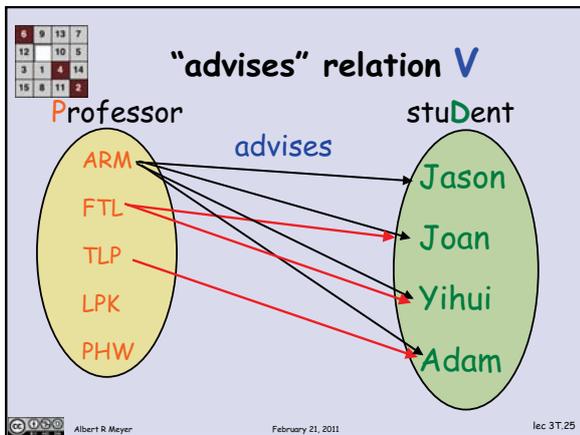
$R^{-1}(J) =$ all the stuDents registered
for some subJect
Every student is registered
for some subject:

$D \subseteq R^{-1}(J)$

(not true: Adam wasn't registered)

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Composing R and V

$$R(V(\{FTL, TLP\})) = R(\{Joan, Yihui, Adam\})$$

Composing R and V

$$R(V(\{FTL, TLP\})) = R(\{Joan, Yihui, Adam\}) = \{6.003, 6.012, 6.004\}$$

$R(V(X))$ = subjects that advisees of profs in X are registered for

Composing R and V

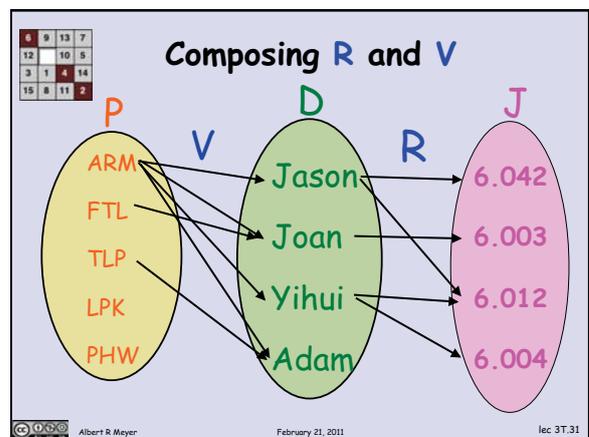
$$(R \circ V)(X) ::= R(V(X))$$

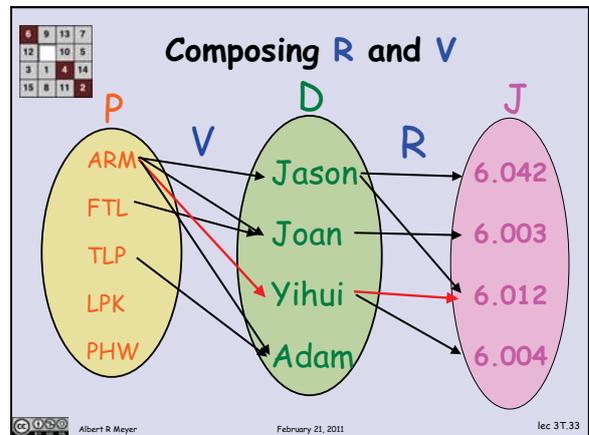
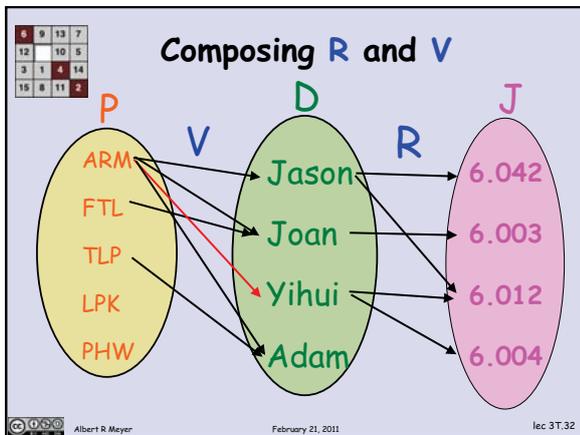
$R \circ V$
is the composition of R and V

Composing R and V

$R \circ V$::= "prof has advisee registered for"

$p(R \circ V)j$::= prof p has an advisee registered in subject j





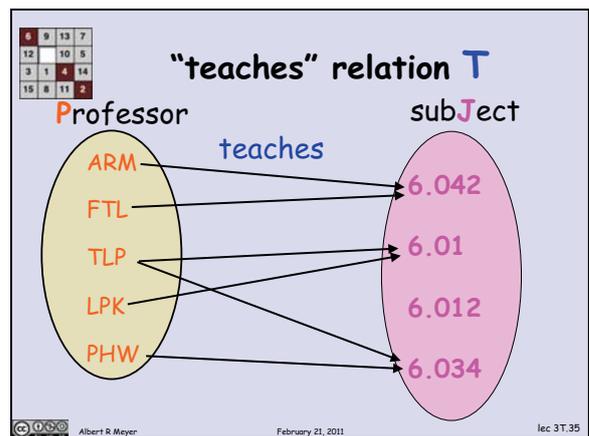
Composing R and V

ARM (R ◦ V) 6.012 because
 ARM V Yihui AND Yihui R 6.012

$p(R \circ V)j$ IFF
 $\exists d \in D. [p V d \text{ AND } d R j]$

note: V, R in reverse order

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set operations on relations

Profs should not teach their advisees:

$\forall p \forall j. \text{NOT}(p(R \circ V)j \text{ AND } pTj)$

$T \cap (R \circ V) = \emptyset$

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set operations on relations

Profs should not teach their advisees:

$\forall p \forall j. \text{NOT}(p(R \circ V)j \text{ AND } pTj)$

$R \circ V \subseteq \overline{T}$

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Binary relations

A binary relation, R , from a set A to a set B associates elements of A with elements of B .



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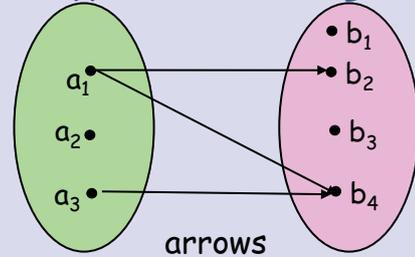
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Binary relation R from A to B

domain A R codomain B



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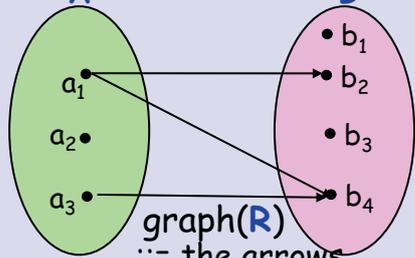
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6	9	13	7
12	10	5	
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Binary relation R from A to B

domain A R codomain B



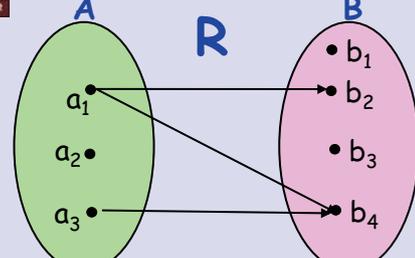
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6	9	13	7
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Binary relation R from A to B



$$\text{graph}(R) = \{(a_1, b_2), (a_1, b_4), (a_3, b_4)\}$$



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6	9	13	7
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range(R)

range(R) ::= elements with arrows coming in
= $R(A)$



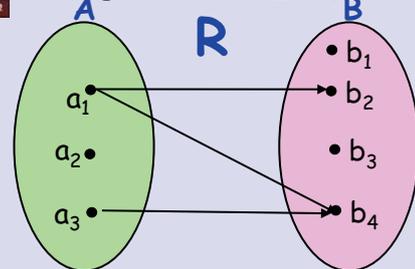
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$$\text{range}(R) = \{b_2, b_4\}$$



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6	9	13	7
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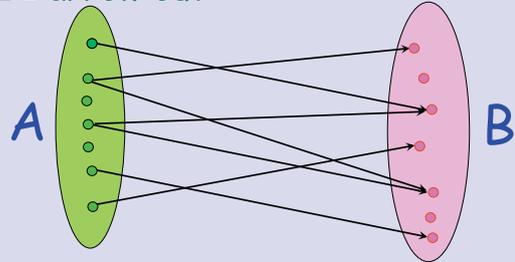
Functions are relations

A function, F , from A to B is a relation which associates each element, a , of A with at most one element of B .
 called $F(a)$

6	9	13	7
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function archery

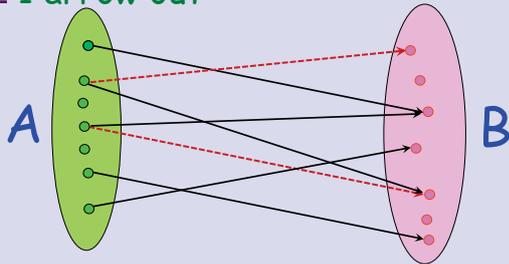
≤ 1 arrow out



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

function archery

≤ 1 arrow out

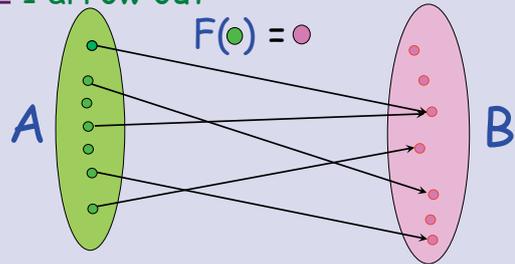


6	9	13	7
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function archery

≤ 1 arrow out

$$F(\bullet) = \bullet$$



6	9	13	7
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Functions are relations

relation $F:A \rightarrow B$ is a function

$$\text{IFF } |F(a)| \leq 1$$

IFF

$$[a F b \text{ AND } a F b'] \text{ IMPLIES } b = b'$$

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