

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Relational Mapping Properties (Archery)

Albert R Meyer February 22, 2012 lec 3W.1

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Binary relation R from A to B

domain A R codomain B

arrows

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| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

function archery

≤ 1 arrow out $F(\bullet) = \bullet$

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| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

archery on relations

$\leq, \geq, = 1$ arrow out $\leq, \geq, = 1$ arrow in

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| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

total relation archery

≥ 1 arrow out

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| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

total relation archery

≥ 1 arrow out

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| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

total relation archery

≥ 1 arrow out

lec 3W.7

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

total relation

R is total iff

$$A = R^{-1}(B)$$

lec 3W.8

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

total & function archery

exactly 1 arrow out

$F(\bullet) = \bullet$

lec 3W.9

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

$$g: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$$

$$g(x, y) ::= \frac{1}{x - y}$$

domain(g) = all pairs of reals
 codomain(g) = all reals

But g is **not total**:
 $g(r, r)$ not defined

lec 3W.10

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

$$g_0: D \rightarrow \mathbb{R}$$

$$g_0(x, y) ::= \frac{1}{x - y}$$

where $D ::= \mathbb{R}^2 - \{(x, y) \mid x = y\}$

g_0, g have the
 same graph, different domains

g_0 is total

lec 3W.11

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

surjection archery

≥ 1 arrow in

lec 3W.12

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

surjection archery

≥ 1 arrow in

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| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

surjection archery

≥ 1 arrow in

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| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

surjection

R is a surjection iff

$$R(A) = B$$

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| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

injection archery

≤ 1 arrow in

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| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

injection archery

≤ 1 arrow in

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| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

injection archery

≤ 1 arrow in

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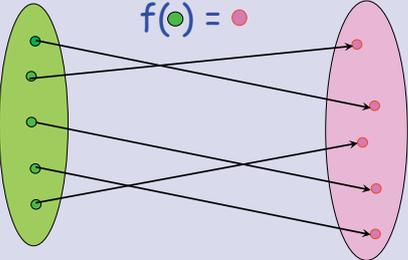
| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

bijection archery

exactly 1 arrow out

exactly 1 arrow in

$f(\bullet) = \bullet$



A

B

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lec 3W.19

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mapping Rule (bij)

A bijection from
A to B implies

$|A| = |B|$

for finite A, B

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lec 3W.20

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