**Slide 1:**

*Mathematics for Computer Science*
*MIT 6.042J/18.062J*

# Proof by Contradiction

Albert R Meyer    February 11, 2013    contradiction.1

**Slide 2:**

**Proof by Contradiction**

Is $\sqrt[3]{1332} \leq 11$?

If so, $1332 \leq 1331$

That's not true, so

$\sqrt[3]{1332} > 11$

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**Slide 3:**

**Proof by Contradiction**

If an assertion implies something false, then the assertion itself must be false!

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**Slide 4:**

**Proof by Contradiction**

Theorem: $\sqrt{2}$ is irrational.

- Suppose $\sqrt{2}$ was rational
- So have n, d integers without common prime factors such that
$$\sqrt{2} = \frac{n}{d}$$
- We will show that n & d are both even. This contradicts no common factor.

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**Slide 5:**

**Proof by Contradiction**

Theorem: $\sqrt{2}$ is irrational.

so can assume

$\sqrt{2} = \frac{n}{d}$

$\sqrt{2}d = n$    QED

$2d^2 = n^2$

So n is even

$n = 2k$

$n^2 = 4k^2$

$2d^2 = 4k^2$

$d^2 = 2k^2$

So d is even

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**Slide 6:**

**Quickie**

Proof assumes that
if $n^2$ is even, then n is even.
Why is this true?

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