Mathematics for Computer Science
MIT 6.042J/18.062J

Predicate Logic, III

∀ ∃ in English

Two Meta-Theorems

Math vs. English

Poet:

"All that glitters is not gold."

∀x. [G(x) IMPLIES NOT(Au(x))]

No: gold glitters like gold!

Math vs. English

Poet:

"All that glitters is not gold."

NOT(∀x. [G(x) IMPLIES Au(x)])

(Poetic license)

Math vs. English

Poet: "There is a season to every purpose under heaven"

∃s ∈ Season ∀p ∈ Purpose. s is for p

Some season, say Summer, is good for all Purposes?

NO, Summer no good for snow shoveling

Math vs. English

Poet: "There is a season to every purpose under heaven"

∃s ∈ Season ∀p ∈ Purpose. s is for p

Poet's meaning flips the quantiers

Math vs. English

Poet: "There is a season to every purpose under heaven"

∀p ∈ Purpose ∃s ∈ Season. s is for p

Poet's meaning flips the quantiers

Math vs. English

Poet: "There is a season to every purpose under heaven"

$\forall p \in \text{Purpose}\ \exists s \in \text{Season}.\ s$ is for p

for snow shoveling, Winter is good
for planting, Spring is good
for leaf watching, Fall is good

Power & Limits of Logic

Two Profound Meta-Theorems about Mathematical Logic

Gödel's Completeness Theorem

Thm 1, good news: only need to know a few axioms & rules to prove all valid formulas.
(in theory; in practice need lots of rules)

Axioms & Inference Rules

Rules are just UG and modus ponens. Most of the valid axioms shown already.

Validity is undecidable

Thm 2, Bad News: there is no procedure to determine whether a quantified formula is valid (in contrast to propositional formulas).

Profound Meta-Theorems

We won't examine these Theorems further. Their proofs usually require half a term in an intro logic course after 6.042. But they are interesting to think about.

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Spring 2015