

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

*Mathematics for Computer Science*  
MIT 6.042J/18.062J

## Predicate Logic, III

### $\forall \exists$ in English

### Two Meta-Theorems

Albert R Meyer, February 17, 2012 lec 2F.1

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### Math vs. English

Poet: "All that <sup>G</sup>glitters is not <sup>Au</sup>gold."

~~$\forall x. [G(x) \text{ IMPLIES NOT}(Au(x))]$~~

**No:** gold glitters like gold!

Albert R Meyer, February 17, 2012 lec 2F.2

6	9	13	7
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### Math vs. English

Poet: "All that glitters is not gold." necessarily

$\text{NOT}(\forall x. [G(x) \text{ IMPLIES } Au(x)])$

(Poetic license)

Albert R Meyer, February 17, 2012 lec 2F.3

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### Math vs. English

Poet: "There is a season to every purpose under heaven"

$\exists s \in \text{Season} \forall p \in \text{Purpose. } s \text{ is for } p$

Some season, say Summer, is good for all Purposes?

**NO**, Summer no good for snow shoveling

Albert R Meyer, February 17, 2012 lec 2F.4

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### Math vs. English

Poet: "There is a season to every purpose under heaven"

$\exists s \in \text{Season} \forall p \in \text{Purpose. } s \text{ is for } p$

Poet's meaning flips the quantiers

Albert R Meyer, February 17, 2012 lec 2F.5

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### Math vs. English

Poet: "There is a season to every purpose under heaven"

$\forall p \in \text{Purpose} \exists s \in \text{Season. } s \text{ is for } p$

Poet's meaning flips the quantiers

Albert R Meyer, February 17, 2012 lec 2F.6

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## Math vs. English

**Poet:** "There is a season to every purpose under heaven"

$\forall p \in \text{Purpose} \exists s \in \text{Season}. s \text{ is for } p$

for snow shoveling, Winter is good  
for planting, Spring is good  
for leaf watching, Fall is good

Albert R Meyer, February 17, 2012 lec 2F.7

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## Power & Limits of Logic

# Two Profound Meta-Theorems about Mathematical Logic

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## Gödel's Completeness Theorem

Thm 1, **good news:** only need to know a few axioms & rules to prove **all** valid formulas.  
(in theory; in practice need lots of rules)

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## Axioms & Inference Rules

Rules are just UG and modus ponens. Most of the valid axioms shown already.

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## Validity is **undecidable**

Thm 2, **Bad News:** there is no procedure to determine whether a quantified formula is valid (in contrast to propositional formulas).

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## Profound Meta-Theorems

We won't examine these Theorems further. Their proofs usually require half a term in an intro logic course after 6.042. But they are interesting to think about.

Albert R Meyer, February 17, 2012 lec 2F.13

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