

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Predicate Logic, II

### Validity & Satisfiability



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Propositional Validity

True for all truth-values.  
Example:

$$(P \text{ IMPLIES } Q) \text{ OR } (Q \text{ IMPLIES } P)$$



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Predicate Calculus Validity

True for all domains and predicates. Example:

$$\forall z.[P(z) \text{ AND } Q(z)] \text{ IMPLIES } [\forall x.P(x) \text{ AND } \forall y.Q(y)]$$



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Proving Validity

$$\forall z.[P(z) \text{ AND } Q(z)] \text{ IMPLIES } [\forall x.P(x) \text{ AND } \forall y.Q(y)]$$

Proof strategy: assume left side is T, then prove right side is T



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Proving Validity

$$\forall z[Q(z) \wedge P(z)] \rightarrow [\forall x.Q(x) \wedge \forall y.P(y)]$$

Proof: Assume left hand side. That is, for all values of  $z$  in the domain,  $Q(z)$  AND  $P(z)$  is true. Suppose  $\text{val}(z) = c$ , an element in the domain. Then  $Q(c)$  AND  $P(c)$  holds, and so  $Q(c)$  by itself holds. But  $c$  could have been any element of the domain. So we conclude  $\forall x.Q(x)$ . (by UG) Similarly conclude  $\forall y.P(y)$ . Therefore,  $\forall x.Q(x)$  AND  $\forall y.P(y)$  QED



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Universal Generalization (UG)

$$\frac{P(c)}{\forall x.P(x)}$$

providing  $c$  does not occur in  $P$



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Similar Example is Not Valid

$\forall z.[P(z) \text{ OR } Q(z)]$  IMPLIES  
 $[\forall x.P(x) \text{ OR } \forall y.Q(y)]$

*Proof: Give counter-model, where left side of IMPLIES is T, but right side is F.*  
 Namely, let domain ::= {1, 2},  
 $Q(z) ::= [z = 1]$ ,  $P(z) ::= [z = 2]$ .

 Albert R Meyer, February 17, 2012 lec 2F.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

DeMorgan's Law for Quantifiers

Another valid formula:

$\text{NOT}(\forall x. P(x))$  IFF  
 $\exists y. \text{NOT}(P(y))$

 Albert R Meyer, February 17, 2012 lec 2F.8

MIT OpenCourseWare  
<http://ocw.mit.edu>

6.042J / 18.062J Mathematics for Computer Science  
Spring 2015

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.