

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

IMPLIES



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IMPLIES.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

IMPLIES

The value of (P IMPLIES Q) is **F** iff
P is **T** and Q is **F**.

Truth Table for IMPLIES

P	Q	P IMPLIES Q
T	T	T
T	F	F
F	T	T
F	F	T



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IMPLIES.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

A True Implication

$(1=-1)$ IMPLIES (I am Pope)
We reasoned *correctly* to
reach the false conclusion



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IMPLIES.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

A True Implication

$(1=-1)$ IMPLIES (I am Pope)
We reasoned *correctly* to
reach the false **conclusion**



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IMPLIES.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

A True Implication

$(1=-1)$ IMPLIES (I am Pope)
 We reasoned *correctly* to reach the false conclusion from the false hypothesis.



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IMPLIES.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

A True Implication

$(1=-1)$ IMPLIES (I am Pope)
 We reasoned *correctly* to reach the false conclusion from the false ~~hypothesis~~.



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IMPLIES.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

A True Implication

$(1=-1)$ IMPLIES (I am Pope)
 The *whole implication* is **true**, even though both conclusion & hypothesis are **false**.



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IMPLIES.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Proving Validity

Instead of truth tables, can try to **prove** valid formulas symbolically using axioms and deduction rules



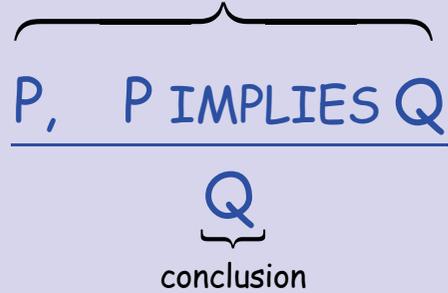
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IMPLIES.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

modus ponens rule
antecedents



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IMPLIES..9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Soundness

A **sound** rule preserves truth:
if all the antecedents are
true in some environment,
then so is the conclusion.



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IMPLIES..10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Soundness

modus ponens is sound:
if **P** is **true**,
and **P IMPLIES Q** is **true**,
then **Q** must be **true**
—by truth table.



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IMPLIES..11

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Soundness & Validity

Lemma: A rule is sound iff
AND{Antecedents}
IMPLIES Conclusion
is **valid**.



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IMPLIES..12

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