Mathematics for Computer Science
MIT 6.042J/18.062J

Sums & Money

Geometric Series

$$G_n = 1 + x + x^2 + \cdots + x^n$$

$$-xG_n = -x - x^2 - \cdots - x^n - x^{n+1}$$

Geometric Series

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Geometric Series

$$G_n = 1 + \cancel{x} + \cancel{x^2} + \cdots + \cancel{x^n}$$

$$-xG_n = -\cancel{x} - \cancel{x^2} - \cdots - \cancel{x^n} - x^{n+1}$$

Geometric Series

$$G_n = 1 + x + x^2 + \cdots + x^n$$
$$-xG_n = \quad - x - x^2 - \cdots - x^n - x^{n+1}$$
$$1 \qquad\qquad - x^{n+1}$$

Geometric Series

$$G_n - xG_n =$$
$$1 \qquad\qquad - x^{n+1}$$

Geometric Series

$$G_n - xG_n = 1 - x^{n+1}$$

$$G_n = \frac{1 - x^{n+1}}{1 - x}$$

Geometric Series

$$G_n = \frac{1 - x^{n+1}}{1 - x}$$

Consider *infinite* sum (series)

$$1 + x + x^2 + \cdots + x^{n-1} + x^n + \cdots = \sum_{i=0}^{\infty} x^i$$

Infinite Geometric Series

$$G_n = \frac{1-x^{n+1}}{1-x}$$

$$\lim_{n\to\infty} G_n = \frac{1-\lim_{n\to\infty} x^{n+1}}{1-x} = \frac{1}{1-x}$$

Infinite Geometric Series

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$$

for $|x| < 1$

The future value of $$

I will pay you $100 in 1 year,
if you will pay me $X now.

The future value of $$

My bank will pay me 3% interest.

define *bankrate*

b ::= 1.03

— bank increases my $$ by this factor in 1 year.

The future value of $$

If I deposit your $X now,
I will have $b·X in 1 year.
So I won't lose money as long as

$$b \cdot X \geq 100$$

$$X \geq \$100/1.03 \approx \$97.09$$

The future value of $$

- $1 in 1 year is worth $0.9709 now.
- $r last year is worth $1 today, where r ::= 1/b.
- So $n paid in 2 years is worth $nr paid in 1 year, and is worth $$nr^2$ today.

The future value of $$

$n paid k years from now is worth $$n \cdot r^k$ today where r ::= 1/bankrate.

Annuities

I pay you $100/year for 10 years,
if you will pay me $Y now.
I *can't lose* if you pay me

$$100r + 100r^2 + 100r^3 + \cdots + 100r^{10}$$
$$= 100r(1 + r + \cdots + r^9)$$
$$= 100r(1-r^{10})/(1-r) = \$853.02$$

Annuities

I pay you $100/year for 10 years, if you will pay me $853.02.

QUICKIE: If bankrates unexpectedly increase in the next few years,

A. You come out ahead
B. The deal stays fair
C. I come out ahead

Manipulating Sums

$$\frac{d}{dx}\left(\sum_{i=0}^{n} x^i\right) = \frac{d}{dx}\left(\frac{1-x^{n+1}}{1-x}\right)$$

$$\sum_{i=0}^{n} ix^{i-1} = \frac{1}{x}\sum_{i=1}^{n} ix^i = \frac{d}{dx}\left(\frac{1-x^{n+1}}{1-x}\right)$$

Manipulating Sums

$$\sum_{i=1}^{n} ix^{i-1} = \frac{x-(n+1)x^{n+1}+nx^{n+2}}{(1-x)^2}$$