

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science
 MIT 6.042J/18.062J

Sums & Money



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Geometric Series

$$G_n = 1 + x + x^2 + \dots + x^n$$

$$-xG_n = -x - x^2 - \dots - x^n - x^{n+1}$$


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6	9	13	7
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Geometric Series

$$G_n = 1 + x + x^2 + \dots + x^n$$

$$-xG_n = -x - x^2 - \dots - x^n - x^{n+1}$$


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6	9	13	7
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Geometric Series

$$G_n = 1 + \overset{\color{red}|}{x} + \overset{\color{red}|}{x^2} + \dots + \overset{\color{red}|}{x^n}$$

$$-xG_n = -\overset{\color{red}|}{x} - \overset{\color{red}|}{x^2} - \dots - \overset{\color{red}|}{x^n} - x^{n+1}$$


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Geometric Series

$$\begin{array}{r}
 G_n = 1 + x + x^2 + \dots + x^n \\
 -xG_n = -x - x^2 - \dots - x^n - x^{n+1} \\
 \hline
 1 \qquad \qquad \qquad -x^{n+1}
 \end{array}$$

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Geometric Series

$$\begin{array}{r}
 G_n - xG_n = \\
 \\
 1 \qquad \qquad \qquad -x^{n+1}
 \end{array}$$

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Geometric Series

$$\begin{array}{r}
 G_n - xG_n = 1 - x^{n+1} \\
 \boxed{G_n = \frac{1 - x^{n+1}}{1 - x}}
 \end{array}$$

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Geometric Series

$$G_n = \frac{1 - x^{n+1}}{1 - x}$$

Consider *infinite* sum (series)

$$1 + x + x^2 + \dots + x^{n-1} + x^n + \dots = \sum_{i=0}^{\infty} x^i$$

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Infinite Geometric Series

$$G_n = \frac{1 - x^{n+1}}{1 - x}$$

$$\lim_{n \rightarrow \infty} G_n = \frac{1 - \lim_{n \rightarrow \infty} x^{n+1}}{1 - x} = \frac{1}{1 - x}$$


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Infinite Geometric Series

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1 - x}$$

for $|x| < 1$



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The future value of \$\$

I will pay you \$100 in 1 year,
if you will pay me \$X now.



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The future value of \$\$

My bank will pay me 3% interest.
define *bankrate*
 $b ::= 1.03$
— bank increases my \$\$ by
this factor in 1 year.



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The future value of \$\$

If I deposit your $\$X$ now,
I will have $\$b \cdot X$ in 1 year.
So I won't lose money as long as

$$b \cdot X \geq 100$$

$$X \geq \$100/1.03 \approx \$97.09$$


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The future value of \$\$

- $\$1$ in 1 year is worth $\$0.9709$ now.
- $\$r$ last year is worth $\$1$ today,
where $r ::= 1/b$.
- So $\$n$ paid in 2 years is worth
 $\$nr$ paid in 1 year, and is worth
 $\$nr^2$ today.



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The future value of \$\$

$\$n$ paid k years from now
is worth $\$n \cdot r^k$ today
where $r ::= 1/\text{bankrate}$.



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Annuities

I pay you $\$100/\text{year}$ for 10 years,
if you will pay me $\$Y$ now.
I can't lose if you pay me

$$100r + 100r^2 + 100r^3 + \dots + 100r^{10}$$

$$= 100r(1 + r + \dots + r^9)$$

$$= 100r(1 - r^{10}) / (1 - r) = \$853.02$$


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Annuities

I pay you \$100/year for 10 years,
if you will pay me \$853.02.

QUICKIE: If bankrates unexpectedly
increase in the next few years,

- A. You come out ahead
- B. The deal stays fair
- C. I come out ahead



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April 10, 2013

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Manipulating Sums

$$\frac{d}{dx} \left(\sum_{i=0}^n x^i \right) = \frac{d}{dx} \left(\frac{1-x^{n+1}}{1-x} \right)$$

$$\sum_{i=0}^n ix^{i-1} = \frac{1}{x} \sum_{i=1}^n ix^i = \frac{d}{dx} \left(\frac{1-x^{n+1}}{1-x} \right)$$



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Manipulating Sums

$$\sum_{i=1}^n ix^{i-1} = \frac{x - (n+1)x^{n+1} + nx^{n+2}}{(1-x)^2}$$



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April 10, 2013

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