

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Generalized Counting Rules



Albert R Meyer, April 19, 2013

genprod.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Generalized Product Rule

lineups of 5 students in class

let $S ::=$ students

say $|S| = 91$ so

~~|lineups of 5 students|~~ NO!

lineups have no repeats:

|seqs in S^5 with no repeats| ?



Albert R Meyer, April 19, 2013

genprod.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Generalized Product Rule

|seqs in S^5 with no repeats|

91 choices for 1st student,

90 choices for 2nd student,

89 choices for 3rd student,

88 choices for 4th student,

87 choices for 5th student

$$= 91 \cdot 90 \cdot 89 \cdot 88 \cdot 87 = \frac{91!}{86!}$$



Albert R Meyer, April 19, 2013

genprod.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Generalized Product Rule

Q a set of length- k sequences

if n_1 possible 1st elements,

n_2 possible 2nd elements

(for each first entry),

n_3 possible 3rd elements

(for each 1st & 2nd entry,...)

then, $|Q| = n_1 \cdot n_2 \cdot \dots \cdot n_k$



Albert R Meyer, April 19, 2013

genprod.4



Division Rule

#6.042 students =

$$\frac{\#6.042 \text{ students' fingers}}{10}$$

Albert R Meyer, April 19, 2013 genprod.5



Division Rule

if function from A to B
 is k -to-1, then

$$|A| = k|B|$$

(generalized Bijection Rule)

Albert R Meyer, April 19, 2013 genprod.6



Counting Subsets

How many size 4 subsets of $\{1,2,\dots,13\}$?
 Let $A ::=$ permutations of $\{1,2,\dots,13\}$
 $B ::=$ size 4 subsets

map $a_1 a_2 a_3 a_4 a_5 \dots a_{12} a_{13} \in A$
 to $\{a_1, a_2, a_3, a_4\} \in B$

Albert R Meyer, April 19, 2013 genprod.7



Counting Subsets

$a_1 a_3 a_2 a_4 a_5 \dots a_{12} a_{13}$ also maps
 to $\{a_1, a_2, a_3, a_4\}$
 so does $a_1 a_3 a_2 a_4 a_{13} \dots a_{12} a_5$
 $4!$ perms $9!$ perms
 all map to same set

$$4! \cdot 9! \text{-to-1}$$

Albert R Meyer, April 19, 2013 genprod.8

6	13	7
12	10	5
3	4	14
15	8	11

Counting Subsets

$$13! = |A| = (4! \cdot 9!) |B|$$

so # of size 4 subsets is

$$\binom{13}{4} ::= \frac{13!}{4!9!}$$



6	13	7
12	10	5
3	4	14
15	8	11

Counting Subsets

m element subsets
of an n element set is

$$\binom{n}{m} ::= \frac{n!}{m!(n-m)!}$$

n choose m



MIT OpenCourseWare
<http://ocw.mit.edu>

6.042J / 18.062J Mathematics for Computer Science
Spring 2015

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.