

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science

MIT 6.042J/18.062J

# Countable Sets



Albert R Meyer, March 4, 2015

countable.1

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## Countable Sets

$A$  is countable iff can be listed  $a_0, a_1, a_2, \dots$

same as  $\mathbb{N}$  bij  $A$  or  $A$  finite

so  $\mathbb{Z}^+$ ,  $\mathbb{Z}$  countable



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countable.2

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## Binary words are countable

$\{0,1\}^*$  ::= finite binary words

list the (empty) string of length 0

list the 2 length-1 bit strings

then list the  $2^2$  length-2 bit strings  
(in binary notation order)

then the  $2^3$  length-3 bit strings

⋮



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countable.3

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## $\mathbb{N} \times \mathbb{N}$ is countable

start with  $(0,0)$

then  $(0,1), (1,0)$

then  $(0,2), (2,0), (1,1)$

then  $(0,3), (3,0), (1,2), (2,1)$

⋮

then all pairs with sum  $n$



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countable.4

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## Proving Countability

Lemma:  $A$  is countable iff  
can list  $A$  allowing repeats:

$$\mathbb{N} \text{ surj } A$$

Corollary:  $A$  is countable iff

$$C \text{ surj } A$$

for some countable  $C$



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countable.5

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## Rationals are countable

map  $(m,n)$  to  $\frac{m}{n}$

$$\underbrace{\mathbb{N} \times \mathbb{N}} \text{ surj } \underbrace{\mathbb{Q}^{\geq 0}}$$

countable so countable



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countable.6

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## Reals are **uncountable**

But  $\{0,1\}^\omega$  and the

real numbers  $\mathbb{R}$

are **not** countable:

next lecture.



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countable.7

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