

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Bookkeeper Rule Multinomial Theorem



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

bookkeeper rule

permutations of the word
bookkeeper ?

- # perms $bo_1o_2k_1k_2e_1e_2e_3r = 10!$
- map perm $o_1be_1o_2k_1rk_2e_2pe_3$ to
obeokrkepe
- 2 o's, 2 k's, 3 e's: $\frac{10!}{2!2!3!}$
map is -to-1



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

bookkeeper rule

permutations of length- n
word with n_1 a's, n_2 b's, ..., n_k z's:

$$\binom{n}{n_1, n_2, \dots, n_k} ::= \frac{n!}{n_1! n_2! \dots n_k!}$$

multinomial coefficient



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

binomial coefficients

binomial a special case:

$$\binom{n}{k} = \binom{n}{k, n-k}$$



6	13	7
12	10	5
3	1	14
15	8	11

multinomials

What is the coefficient of
 EMS^3TY
 in the expansion of
 $(E + M + S + T + Y)^7$?

The number of ways to
 rearrange the letters in
 the word
 $SYSTEMS$



Albert R Meyer, April 22, 2013

bookkeeper.6

6	13	7
12	10	5
3	1	14
15	8	11

applying the BOOKKEEPER rule

What is the coefficient of
 EMS^3TY
 in the expansion of
 $(E + M + S + T + Y)^7$?

$$\binom{7}{1, 1, 3, 1, 1}$$



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bookkeeper.7

6	13	7
12	10	5
3	1	14
15	8	11

multinomial coefficients

What is the coefficient of
 BA^3N^2
 in the expansion of
 $(B + A + N)^6$?

The number of ways to
 rearrange the letters in
 the word
 $BANANA$



Albert R Meyer, April 22, 2013

bookkeeper.10

6	13	7
12	10	5
3	1	14
15	8	11

multinomial coefficients

What is the coefficient of
 BA^3N^2
 in the expansion of
 $(B + A + N)^6$?

$$\binom{6}{1, 3, 2}$$



Albert R Meyer, April 22, 2013

bookkeeper.11

6	13	7
12	10	5
3	1	14
15	8	11
2	4	9

multinomial coefficients

Take 14 mile walk including 3 Northward miles, 4 Southward, 5 Eastward and 3 Westward. How many different walks?

= #rearrangements of

$$N^3 S^4 E^5 W^2$$

$$= \binom{14}{3, 4, 5, 2}$$



Albert R Meyer, April 22, 2013

bookkeeper.12

6	13	7
12	10	5
3	1	14
15	8	11
2	4	9

multinomial coefficients

What is the coefficient of

$$X_1^{r_1} X_2^{r_2} X_3^{r_3} \dots X_k^{r_k}$$

in the expansion of

$$(X_1 + X_2 + X_3 + \dots + X_k)^n ?$$

$$\binom{n}{r_1, r_2, r_3, \dots, r_k}$$



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bookkeeper.13

6	13	7
12	10	5
3	1	14
15	8	11
2	4	9

The Multinomial Formula

$$(X_1 + X_2 + \dots + X_k)^n =$$

$$\sum_{r_1 + \dots + r_k = n} \binom{n}{r_1, r_2, r_3, \dots, r_k} X_1^{r_1} X_2^{r_2} X_3^{r_3} \dots X_k^{r_k}$$



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bookkeeper.14

6	13	7
12	10	5
3	1	14
15	8	11
2	4	9

multinomial coefficients

$$\binom{n}{r_1, r_2, r_3, \dots, r_k}$$

$$:= 0 \quad \text{if } r_1 + r_2 + \dots + r_k \neq n$$



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bookkeeper.15

6	8	13	7
12		10	5
3	1	4	14
15	9	11	2

Preceding slides adapted from:

Great Theoretical Ideas In Computer Science

Carnegie Mellon Univ., CS 15-251, Spring 2004

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Applied Combinatorics, by Alan Tucker



Albert R Meyer, April 22, 2013

bookkeeper.17

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