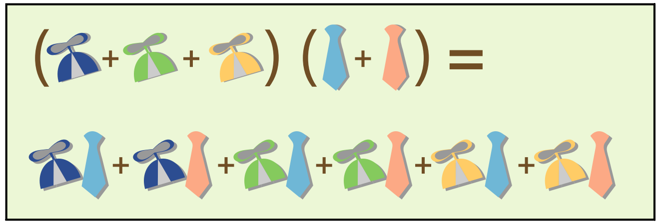
Mathematics for Computer Science

MIT 6.042J/18.062J

Binomial Theorem

Polynomials Express Choices & Outcomes



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Products of Sums = Sums of Products

expression for c_k?

$$(1+X)^n =$$

$$c_0 + c_1 X + c_2 X^2 + \ldots + c_n X^n$$

binomial expressions

$$(1+X)^0 = 1$$

$$(1+X)^1 = 1 + 1X$$

$$(1+X)^2 = 1 + 2X + 1X^2$$

$$(1+X)^3 = 1 + 3X + 3X^2 + 1X^3$$

$$(1+X)^4 = 1 + 4X + 6X^2 + 4X^3 + 1X^4$$

expression for c_k?

$(1+X)^n$

$= \underbrace{(1+X)(1+X)(1+X)(1+X)\ldots(1+X)}_{n \text{ times}}$

multiplying gives 2^n product terms:

$11\cdots1 + X11X\cdots X1 + 1XX\cdots 1X1 + \cdots + XX\cdots X$

a term corresponds to selecting 1 or X from each of the n factors

expression for c_k?

$(1+X)^n$

$= \underbrace{(1+X)(1+X)(1+X)(1+X)\ldots(1+X)}_{n \text{ times}}$

the X^k coeff, c_k, is # terms with exactly k X's selected

$$c_k = \binom{n}{k}$$

The Binomial Formula

$$(1+X)^n = \binom{n}{0} + \binom{n}{1}X + \binom{n}{2}X^2 + \ldots + \binom{n}{k}X^k + \ldots + \binom{n}{n}X^n$$

binomial expression

binomial coefficients

The Binomial Formula

$$(X+Y)^n = \binom{n}{0}Y^n + \binom{n}{1}XY^{n-1} + \binom{n}{2}X^2Y^{n-2} + \ldots + \binom{n}{k}X^kY^{n-k} + \ldots + \binom{n}{n}X^n$$

The Binomial Formula

$$(X+Y)^n = \sum_{k=0}^{n} \binom{n}{k} X^k Y^{n-k}$$