

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Bijections for Counting



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bijecount.1

6	9	13	7
12		10	5
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Example: Counting Passwords

Password conditions:

- characters are digits & letters
- between 6 & 8 characters long
- starts with a letter
- case sensitive



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bijecount.2

6	9	13	7
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Counting Passwords

$L ::= \{a, b, \dots, z, A, B, \dots, Z\}$

$D ::= \{0, 1, \dots, 9\}$

$P_n ::=$ length n words
starting w/letter

$$= L \times (L \cup D)^{n-1}$$



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bijecount.3

6	9	13	7
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Counting Passwords

$$\begin{aligned} & |L \times (L \cup D)^{n-1}| \\ &= |L| \cdot |(L \cup D)|^{n-1} \\ &= |L| \cdot (|L| + |D|)^{n-1} \\ &= 52 \cdot (52 + 10)^{n-1} \end{aligned}$$



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bijecount.4

 **Counting Passwords**
 set of passwords:
 $P ::= P_6 \cup P_7 \cup P_8$
 $|P| = |P_6| + |P_7| + |P_8|$
 $= 52 \cdot (62^5 + 62^6 + 62^7)$
 $\approx 19 \cdot 10^{14}$

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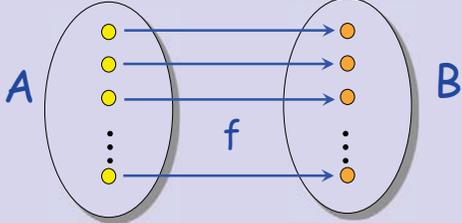
 **# 4-digit nums w/ \geq one 7**
 cases by 1st occurrence of 7:
 x : any digit o : any digit $\neq 7$
 $7xxx$ or $o7xx$ or $oo7x$ or $ooo7$
 $10^3 + 9 \cdot 10^2 + 9^2 \cdot 10 + 9^3$
 $= 3439$

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 at least one 7: another way
 $|4\text{-digit nums w/ } \geq \text{one } 7|$
 $= |4\text{-digit nums}|$
 $\quad - |those w/ \text{no } 7|$
 $= 10^4 - 9^4 = 3439$

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 **Mapping Rule: Bijections**
 If f is a bijection from A to B ,
 then $|A| = |B|$



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Bijection: $\mathcal{P}(A)$ and Binary Strings

A: $\{a_1, a_2, a_3, a_4, a_5, \dots, a_n\}$
 subset: $\{a_1, a_3, a_4, \dots, a_n\}$
 string: 1 0 1 1 0 ... 1

This is a bijection, so

$\underbrace{|\text{n-bit binary strings}|}_{2^n} = |\mathcal{P}(A)|$



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Size of $\mathcal{P}(A)$

$|\mathcal{P}(A)| = 2^{|A|}$



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Counting Doughnut Selections

From 5 kinds of doughnuts
 select a dozen.

let $A ::=$ all selections of
 12 doughnuts

$\underbrace{00}_{\text{chocolate}}$
 $\underbrace{\text{(none)}}_{\text{lemon}}$
 $\underbrace{000000}_{\text{sugar}}$
 $\underbrace{00}_{\text{glazed}}$
 $\underbrace{00}_{\text{plain}}$



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Counting Doughnut Selections

$B ::=$ 16-bit words with four 1's

0011000000100100

$\underbrace{001}_{\text{chocolate}}$
 $\underbrace{1000000}_{\text{sugar}}$
 $\underbrace{100}_{\text{glazed}}$
 $\underbrace{100}_{\text{plain}}$



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6	9	13	7
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3	4	8	14
15	2	11	4

Bijection from A to B

c chocolate, l lemon, s sugar, g glazed, p plain

maps to

$0^c 10^l 10^s 10^g 10^p$

so

$$|A| = |B|$$



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April 17, 2013

bijectioncount.16

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