

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science  
MIT 6.042J/18.062J

# Trees

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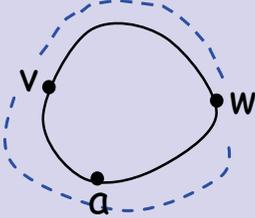
Albert R Meyer, April 8, 2013

tree-def.1

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Cycles in simple graphs

A **cycle** is a closed walk of **length**  $> 2$  that doesn't cross itself:



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tree-def.2

6	9	13	7
12	10	5	
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## Cycles in simple graphs

**length**  $> 2$  implies that going back & forth over an edge is **not** a cycle



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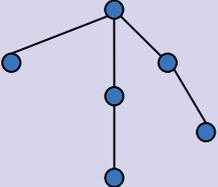
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tree-def.3

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Trees

A **tree** is a connected graph with **no cycles**.



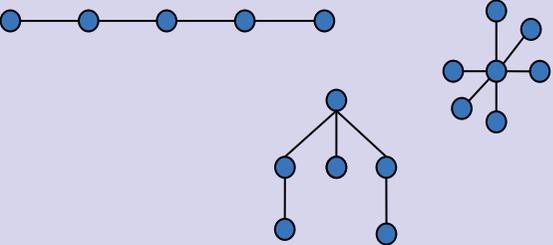
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tree-def.4

6	9	13	7
12	10	5	
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## More Trees



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tree-def.5

6	9	13	7
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## Come up all the time

- family trees
- search trees
- game trees
- parse trees
- spanning trees

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tree-def.6

6	9	13	7
12	10	5	
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## Lots of kinds

- rooted
- ordered
- binary
- complete
- directed

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tree-def.7

6	9	13	7
12	10	5	
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15	8	11	2

## Focus on "pure" trees

unordered,  
unrooted,  
undirected

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tree-def.9

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Cut Edges

An edge is a **cut edge** if removing it from the graph disconnects two vertices.

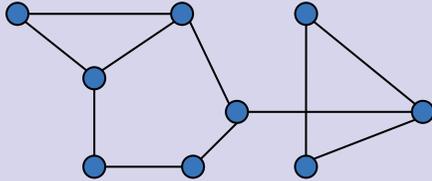


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tree-def.10

6	9	13	7
12	10	5	
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## Cut Edges

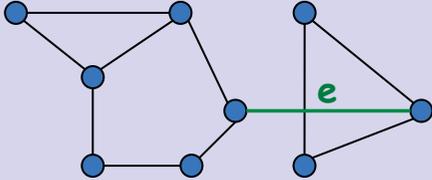



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tree-def.11

6	9	13	7
12	10	5	
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## Cut Edges



**e** is a cut edge

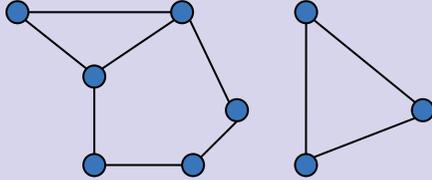


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tree-def.12

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Cut Edges



deleting **e** gives two components

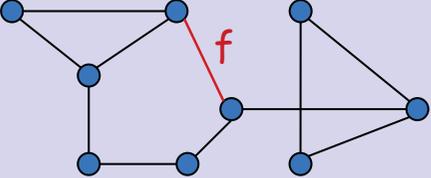


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tree-def.13

6	9	13	7
12	10	5	
3	1	4	14
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### Cut Edges



**f** is not a cut edge

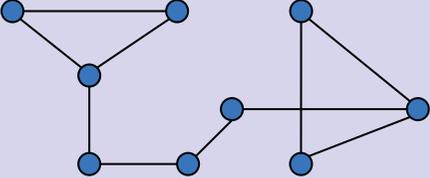
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tree-def.14

6	9	13	7
12	10	5	
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### Cut Edges



still connected with  
edge **f** deleted

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tree-def.15

6	9	13	7
12	10	5	
3	1	4	14
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### Cut Edges

So a connected graph is  
2-edge connected iff  
it has no cut edge.

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tree-def.16

6	9	13	7
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### Cut Edges and Cycles

*Lemma:* An edge is a  
not a cut edge iff  
it is on a cycle.

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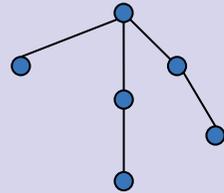
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tree-def.17

6	9	13	7
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## alternative tree definition

A **tree** is a connected graph with **every edge a cut edge**.



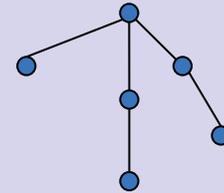
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tree-def.18

6	9	13	7
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## alternative tree definition

A **tree** is a connected graph that is **edge-minimal**.



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tree-def.19

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## more equivalent definitions

- connected graph with  $n$  vertices and  $n-1$  edges
- an **edge-maximal** acyclic graph
- graph with a **unique** path between **any 2** vertices



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tree-def.22

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6.042J / 18.062J Mathematics for Computer Science  
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