Mathematics for Computer Science
MIT 6.042J/18.062J

Introduction to Probability Theory The Tree Model

Counting in Probability

What is the *probability* of getting

exactly two jacks

in a poker hand?

Counting in Probability

Outcomes: $\binom{52}{5}$ 5-card hands

Event: $\binom{4}{2}\binom{52-4}{3}$ hands w/2Jacks

$$\Pr[2\text{ Jacks}] ::= \frac{\binom{4}{2}\binom{52-4}{3}}{\binom{52}{5}} \approx 0.04$$

Probability: 1st Idea

- A set of basic experimental outcomes
- A subset of outcomes is an event
- The probability of an event:

$$\Pr[\text{event}] ::= \frac{\#\text{ outcomes in event}}{\text{total }\#\text{ outcomes}}$$

The Monty Hall Game

Applied Probability:

Let's Make A Deal

(1970's TV Game Show)

Monty Hall Webpages

Image of three doors removed due to copyright restrictions.

http://www.letsmakeadeal.com

Monty Hall Webpages

Image of Monty Hall and contestants removed due to copyright restrictions.

Monty Carol Merill

http://www.letsmakeadeal.com

The Monty Hall Game

- goats behind two doors
- prize behind third door
- contestant picks a door
- Monty reveals a goat behind an unpicked door
- Contest sticks, or switches to the other unopened door

Analyzing Monty Hall

Marilyn Vos Savant explained Game in magazine -- bombarded by letters (even from PhD's) debating:

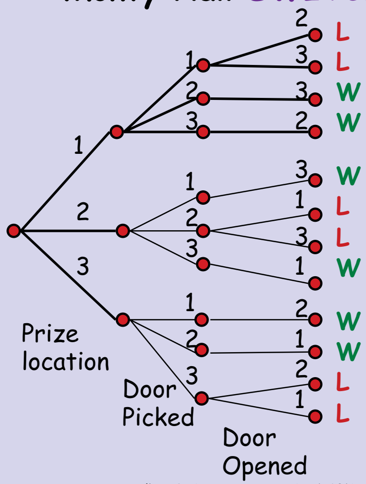
1) sticking & switching equally good
2) switching better

Analyzing Monty Hall

Determine the outcomes.

-- using a tree of possible steps can help

Monty Hall SWITCH strategy



SWITCH
Wins: 6

Lose: 6

Monty Hall STICK strategy

Win by sticking
iff
Lose by switching

STICK
Lose: 6

Wins: 6

Analyzing Monty Hall

A false conclusion:
sticking and switching have same # winning outcomes, so probability of winning is the same for both: 1/2.

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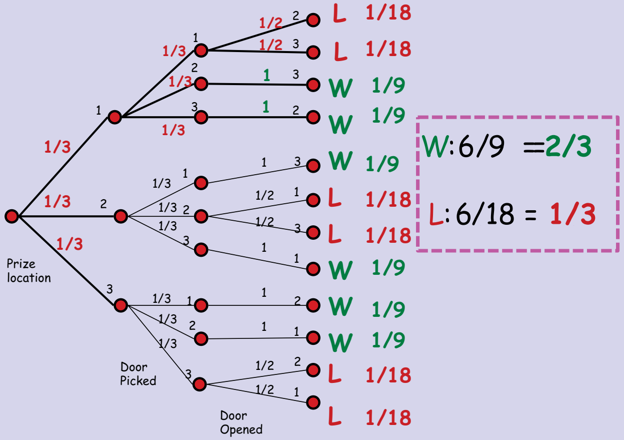
Analyzing Monty Hall

What's wrong?
Let's look at the outcome tree more carefully.

Monty Hall SWITCH strategy



Monty Hall SWITCH strategy

Pr[switch wins]

$$= \frac{2}{3}$$

Probability: 2nd Idea

Outcomes may have differing probabilities!

Not always uniform.

Finding Probability

Intuition is important but dangerous.

Stick with 4-part method:

1. Identify outcomes

Finding Probability

Intuition is important but dangerous.

Stick with 4-part method:

1. Identify outcomes (tree helps)
2. Identify event (winning)
3. Assign outcome probabilities
4. Compute event probabilities