

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science
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Total Expectation

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lec 12F.1

6	9	13	7
12	10	5	
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Law of Total Expectation

good for reasoning by cases

Def: conditional expectation

$$E[R | A] ::= \sum v \cdot \text{pr}[R = v | A]$$

$$E[R] = E[R | A] \cdot \text{Pr}[A] + E[R | \bar{A}] \cdot \text{Pr}[\bar{A}]$$

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6	9	13	7
12	10	5	
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Law of Total Expectation

More generally, many cases:

$$E[R] = E[R | A_1] \cdot \text{Pr}[A_1] + E[R | A_2] \cdot \text{Pr}[A_2] + \dots + E[R | A_n] \cdot \text{Pr}[A_n] + \dots$$

when $\{A_i\}$ partitions the sample space

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Expected #Heads

Let $e(n) ::=$ expected #H's in n flips.

$$= 1 + e(n-1) \quad \text{if 1st flip H}$$

$$= e(n-1) \quad \text{if 1st flip T}$$

by Total Expectation:

$$e(n) = [1 + e(n-1)] \cdot p + e(n-1) \cdot q$$

$$e(n) = e(n-1) + p = e(n-2) + 2p = \dots$$

$$= e(0) + np = np$$

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lec 12F.4

6	9	13	7
12		10	5
3	1	4	14
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Expected #Heads

Let $e(n) ::=$ expected #H's in n flips.

$$= 1 + e(n-1) \quad \text{if 1st flip H}$$

$$= e(n-1) \quad \text{if 1st flip T}$$

by Total Expectation:

$$e(n) = [1 + e(n-1)] \cdot p + e(n-1) \cdot q$$

$$e(n) = e(n-1) + p = np = E[B_{n,p}]$$



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