

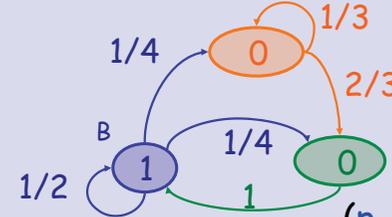
6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Stationary Distributions



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12		10	5
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Distribution Over Nodes



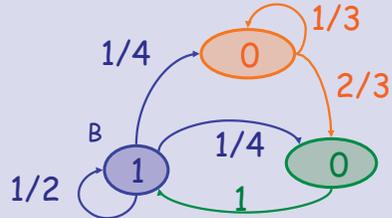
Suppose you start at B: (p_B, p_O, p_G)
 $(1, 0, 0)$

What are p'_B, p'_O, p'_G after 1 step?



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Distribution Over Nodes

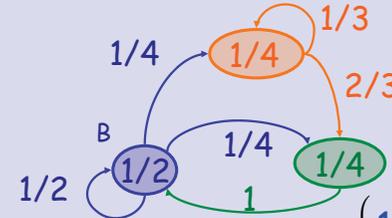


Dist after 1 step: (p'_B, p'_O, p'_G) ,
only get places from B, $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 4 \end{pmatrix}$
so



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Distribution Over Nodes



Dist after 1 step: $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 4 \end{pmatrix}$
Dist after 2 steps: (p''_B, p''_O, p''_G)



Distribution Over Nodes

Dist after 1 step:

$$p''_O = \Pr[B \text{ to } O | \text{at } B] \cdot p'_B + \Pr[O \text{ to } O | \text{at } O] \cdot p'_O + \Pr[G \text{ to } O | \text{at } G] \cdot p'_G$$

$$\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4} \right)$$

stationary.7

Distribution Over Nodes

Dist after 1 step:

$$p''_O = \begin{matrix} \cdot \frac{1}{2} \\ + \cdot \frac{1}{4} \\ + \cdot \frac{1}{4} \end{matrix} = \frac{5}{24}$$

stationary.8

Distribution Over Nodes

distribution after 2 steps:

$$(p''_B, p''_O, p''_G)$$

$$\left(\frac{1}{2}, \frac{5}{24}, \frac{7}{24} \right)$$

stationary.9

Linear Algebra

The **edge probability matrix** for a random walk graph is the same as the adjacency matrix, using edge probabilities instead of zeroes and ones.

stationary.11

6	13	7
12	10	5
3	1	4
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Linear Algebra

the edge probability matrix

$M ::=$

$$\begin{pmatrix} \Pr[B \rightarrow B] & \Pr[B \rightarrow O] & \Pr[B \rightarrow G] \\ \Pr[O \rightarrow B] & \Pr[O \rightarrow O] & \Pr[O \rightarrow G] \\ \Pr[G \rightarrow B] & \Pr[G \rightarrow O] & \Pr[G \rightarrow G] \end{pmatrix}$$



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stationary.14

6	13	7
12	10	5
3	1	4
15	8	11

Linear Algebra

the edge probability matrix

$M ::=$

$$\begin{pmatrix} 1/2 & 1/4 & 1/4 \\ \Pr[O \rightarrow B] & \Pr[O \rightarrow O] & \Pr[O \rightarrow G] \\ \Pr[G \rightarrow B] & \Pr[G \rightarrow O] & \Pr[G \rightarrow G] \end{pmatrix}$$



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stationary.15

6	13	7
12	10	5
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15	8	11

Linear Algebra

the edge probability matrix

$$M ::= \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 0 & 1/3 & 2/3 \\ 1 & 0 & 0 \end{pmatrix}$$



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stationary.17

6	13	7
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Linear Algebra

distribution update is
vector/matrix multiplication

$$\begin{aligned} (p_B, p_O, p_G) \cdot M \\ = (p'_B, p'_O, p'_G) \end{aligned}$$



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stationary.18

Distribution Over Nodes

distribution after t steps?
...and as $t \rightarrow \infty$?

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Linear Algebra

$$(p_B, p_O, p_G) \cdot M^t = (p_B^t, p_O^t, p_G^t)$$

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Stationary Distribution

distribution (p_B, p_O, p_G) is **stationary** if next-step distribution is the same.
What is a stationary dist. here?

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Finding Stationary Dist.

$$p_B = p_B' = (1/2)p_B + 1p_G$$

$$p_O = p_O' = (1/4)p_B + (1/3)p_O$$

$$p_G = p_G' = (1/4)p_B + (2/3)p_O$$

$$p_B + p_O + p_G = 1$$

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Finding Stationary Dist.

solving for (p_B, p_O, p_G) : $\begin{pmatrix} \frac{8}{15} & \frac{3}{15} & \frac{4}{15} \end{pmatrix}$

stationary.23

Linear Algebra

Find stationary dist vector \vec{s} by solving:

$$\vec{s} \cdot M = \vec{s}$$

$$\sum s_i = 1$$

stationary.24

Stationary Difficulties

does not converge to stable distribution

uncountably many stable distributions

stationary.25

Questions on Stationary Dist

- \exists stationary dist? **Yes** (if graph finite)
- unique? **Sometimes**
- converge to it from any start? **Sometimes**
- -How quickly? **Varies**

stationary.26

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