

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
 MIT 6.042J/18.062J

Representing Partial Orders

Albert R Meyer March 22, 2013

rep-po.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

proper subset relation

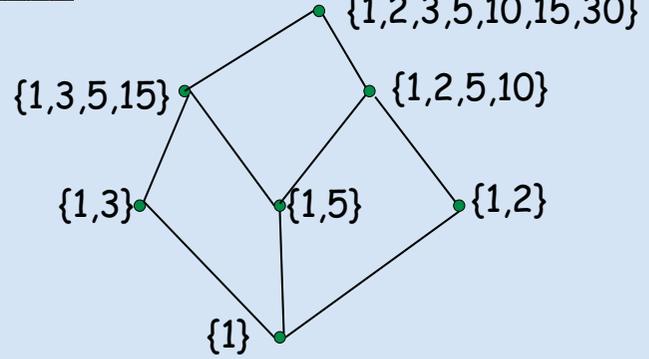
$A \subset B$ means
 B has everything
 that A has
 and more: $B \not\subset A$

Albert R Meyer March 22, 2013

rep-po.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

proper subset relation

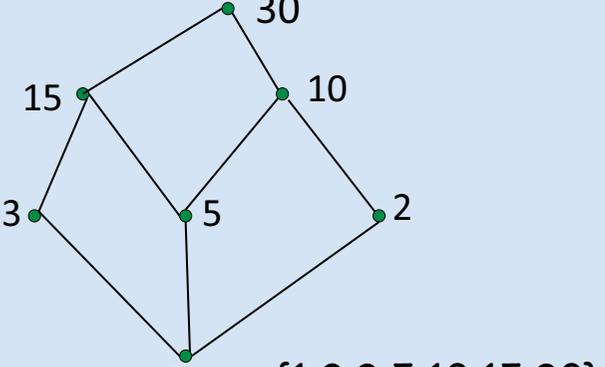


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rep-po.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

partial order: properly divides



1 on {1,2,3,5,10,15,30}

Albert R Meyer March 22, 2013

rep-po.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

same shape

as \subset example

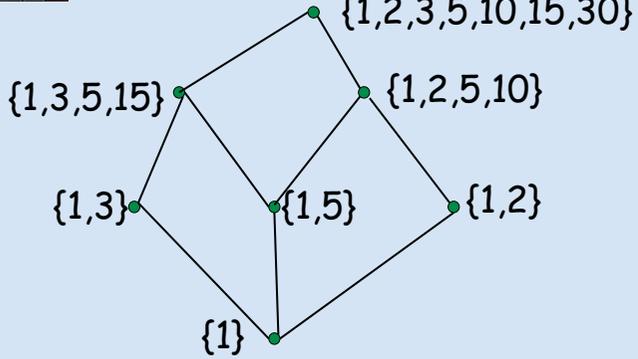


Albert R Meyer March 22, 2013

rep-po.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

proper subset

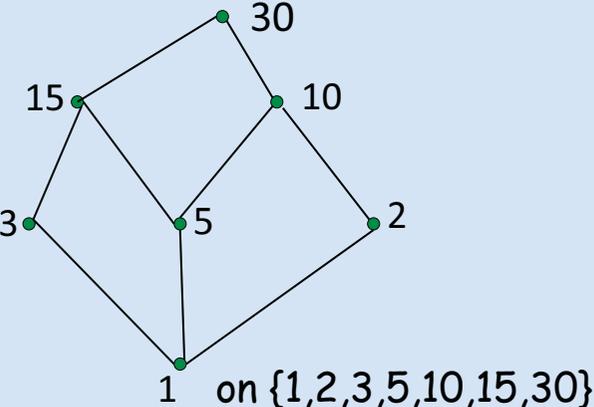



Albert R Meyer March 22, 2013

rep-po.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

partial order: properly divides



1 on {1,2,3,5,10,15,30}



Albert R Meyer March 22, 2013

rep-po.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

same shape

as \subset example

isomorphic



Albert R Meyer March 22, 2013

rep-po.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Isomorphism

All that matters are the **connections**: graphs with the same connections are **isomorphic**

 Albert R Meyer March 19, 2012 rep-po.9

6	9	13	7
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Isomorphism

two graphs are **isomorphic** when there is an **edge-preserving bijection** of their vertices.

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Formal Def of Graph Isomorphism

G_1 isomorphic to G_2 iff

\exists bijection $f: V_1 \rightarrow V_2$ with

$u \rightarrow v$ in E_1 IFF $f(u) \rightarrow f(v)$ in E_2

 Albert R Meyer March 19, 2012 rep-po.11

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p.o. represented by \subset

Theorem: Every strict partial order is isomorphic to a collection of subsets partially ordered by \subset .

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p.o. isomorphic to \subset

proof: map element, a , to the set of elements below it.
 a maps to $\{b \in A \mid bRa \text{ OR } b=a\}$



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rep-po.13

6	9	13	7
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p.o. isomorphic to \subset

proof: map element, a , to the set of elements below it.
 a maps to $\{b \in A \mid bRa \text{ OR } b=a\}$

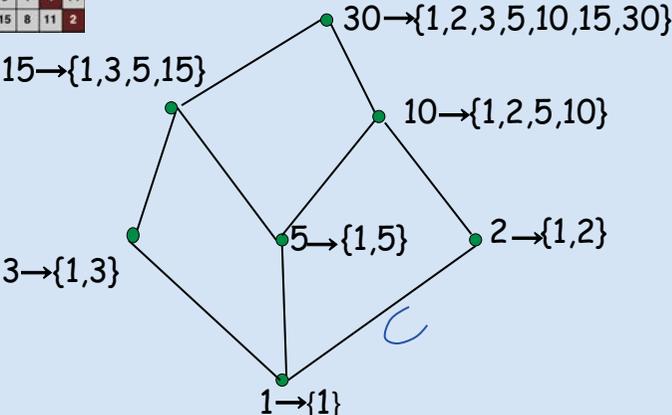
$$f(a) ::= R^{-1}(a) \cup \{a\}$$


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rep-po.14

6	9	13	7
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subsets from divides



$15 \rightarrow \{1, 3, 5, 15\}$

$3 \rightarrow \{1, 3\}$

$1 \rightarrow \{1\}$

$5 \rightarrow \{1, 5\}$

$2 \rightarrow \{1, 2\}$

$10 \rightarrow \{1, 2, 5, 10\}$

$30 \rightarrow \{1, 2, 3, 5, 10, 15, 30\}$



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rep-po.15

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