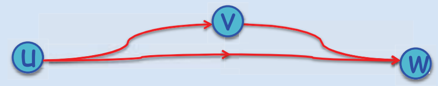
Mathematics for Computer Science

MIT 6.042J/18.062J

Partial Orders

Walks in digraph G

walk from u to v and from v to w



implies walk from u to w

Walks in digraph G

walk from u to v and from v to w, implies walk from u to w:

$u\ G^+\ v$ AND $v\ G^+\ w$
IMPLIES $u\ G^+\ w$

Walks in digraph G

transitive relation R:

$u\ R\ v$ AND $v\ R\ w$
IMPLIES $u\ R\ w$

G^+ is transitive

transitivity

Theorem:

R is a transitive iff

R = G+ for some digraph G

Paths in DAG D

pos length path from u to v implies no path from v to u

$u\ D^+\ v$ IMPLIES NOT$(v\ D^+\ u)$

Paths in DAG D

asymmetric relation R:

u R v IMPLIES NOT(v R u)

D^+ is asymmetric

strict partial orders

transitive & asymmetric

strict partial orders

examples:

- $\subset$ on sets
- "indirect prerequisite" on MIT subjects
- less than, $<$, on real numbers

strict partial orders

Theorem:

R is a SPO iff
R = D+ for some DAG D

linear orders

Given any two elements, one will be "bigger than" the other one.

linear orders

basic example:

$<$ or $\leq$ on the Reals:
if $x \neq y$, then either
$x < y$ OR $y < x$

linear orders

R is linear:

no incomparable elements

if $x \neq y$, then either

$x \, R \, y$ OR $y \, R \, x$

linear orders

The whole partial order is a chain

linear orders

A topological sort turns a partial order into a linear order …in a way that is consistent with the partial order

weak partial orders

same as a strict partial order R, except that

$a \, R \, a$ always holds

examples:

$\leq$ is weak p.o. on $\mathbb{R}$

$\subseteq$ is weak p.o. on sets

reflexivity

relation R on set A
is reflexive iff
a R a for all $a \in A$

G^* is reflexive

antisymmetry

binary relation R is
antisymmetric iff
it is asymmetric
except for a R a case.

A/Antisymmetry

minor difference:
whether aRa is allowed

never sometimes

antisymmetry

antisymmetric relation R:

u R v IMPLIES NOT(v R u)
for $u \neq v$

D^* is antisymmetric for
DAG D

weak partial orders

transitive,
antisymmetric &
reflexive

weak partial orders

Theorem:

R is a WPO iff
R = D* for some
DAG D