



• *Mathematics for Computer Science*
MIT 6.042J/18.062J

Optimal Stable Matching



Albert R. Meyer, 2015 September 29, 2015 optimal.1



Mating Ritual

Who does better,
boys or girls?
Girls' suitors get better,
and boys' sweethearts
get worse



Albert R. Meyer, 2015 September 29, 2015 optimal.2



Mating Ritual

Who does better,
boys or girls?
Girls' suitors get better,
and boys' sweethearts
get worse, so girls do
better? **No!**



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Feasible spouses

Nicole and Keith are
feasible spouses when
they are married in
some set of stable
marriages.



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Optimal spouses

Nicole is **optimal** for
Keith when she is
Keith's **highest ranked**
feasible spouse



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September 29, 2015

optimal.6

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Boys **win** the Ritual
In Mating Ritual:
Every boy marries his
optimal wife.



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Boys **win** the Ritual

Follows from **Invariant**
No girl has rejected
a feasible spouse



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Boys **win** the Ritual

Assume the **Invariant**.
Boy's wife is top-listed girl
on Wedding Day.



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Boys **win** the Ritual

Assume the **Invariant**.

Boy's wife is top-listed girl on Wedding Day. Deleted girls not feasible, so wife is highest ranked feasible.

optimal wife



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Verify the **Invariant**

No girl has rejected a feasible spouse



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optimal.11

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Verify the **Invariant**

Suppose Invariant holds when Nicole rejects Bob. Must show that Bob is not feasible for Nicole.



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optimal.12

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Verify the **Invariant**

Nicole rejects Bob
...because Tom is serenading her and she prefers Tom to Bob.



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Verify the **Invariant**
By **Invariant**, Tom's
feasible wives still on his
list.



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optimal.15

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Verify the **Invariant**
By **Invariant**, Tom's
feasible wives still on his
list. Nicole is top of list,
so ranks above all Tom's
other feasible.



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Verify the **Invariant**
So Nicole cannot stably
marry Bob, because



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Verify the **Invariant**
So Nicole cannot stably
marry Bob, because **she and
Tom would be rogue:**
She prefers Tom to Bob,
and Tom would prefer her to
whoever his wife is.



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optimal.19

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Verify the **Invariant**

So rejected suitor Bob
is indeed not feasible:
Invariant is preserved!



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optimal.20

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Girls **lose** the Ritual

Easier argument shows
each girl gets worst
possible spouse.



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optimal.21

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Boys win, Girls lose
In Mating Ritual:

Every boy marries his
optimal wife.

Every girl marries her
pessimal husband.



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optimal.22

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage

- More questions, rich theory
- other stable marriages possible? - can be **many**
 - do better by lying?
boys **-No!** girls **-Yes!**



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optimal.23

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