Mathematics for Computer Science
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Mutually Independent Events

Mutual Independence

Events $A_1, A_2, \ldots, A_n$ are mutually independent when the probability that A_i occurs

Mutual Independence

Events $A_1, A_2, \ldots, A_n$ are mutually independent when the probability that A_i occurs is unchanged by which other ones occur.

Mutual Independence

Example: Successive coin flips

H_i ::= [i^{th} flip is Heads]

What happens on the 5^{th} flip is independent of what happens on the 1^{st}, 4^{th}, or 7^{th} flip:

$$\Pr[H_5] = \Pr[H_5 \mid H_1 \cap H_4 \cap \overline{H_7}]$$

Mutual Independence

Events $A_1, A_2, \ldots, A_n$ are **mutually independent** when

$$\Pr[A_i] = \Pr[A_i \mid A_j \cap A_k \cap \cdots \cap A_m]$$

$$(i \neq j, k, \ldots, m)$$

Mutual Independence

Events $A_1, A_2, \ldots, A_n$ are **mutually independent** when

$$\Pr[A_i \cap A_j \cap \cdots \cap A_m] =$$

$$\Pr[A_i] \cdot \Pr[A_j] \cdots \Pr[A_m]$$

Pairwise Independence

Example: Flip a fair coin twice

H_1 ::= [Head on 1st flip]

H_2 ::= [Head on 2nd flip]

O ::= [Odd # Heads]

Claim: O is independent of H_1

Pairwise Independence

Example: Flip a fair coin twice

O is independent of H_1:

$$O = \{HT, TH\}, \quad \Pr[O] = 1/2$$

$$O \cap H_1 = \{HT\}, \quad \Pr[\{HT\}] = 1/4$$

$$\Pr[O \cap H_1] = 1/4 = \Pr[O] \cdot \Pr[H_1]$$

✓

Not Mutually Independent

Example: Flip a fair coin twice

But O, H_1, H_2 not mutually independent:

$$\Pr[O \mid H_1 \cap H_2] = 0 \neq \Pr[O]$$

k-way Independence

Example: Flip a fair coin k times

H_i ::= [Head on i^{th} flip]

O ::= [Odd # Heads]

Claim: Any set of k of these events are mutually independent, but all k+1 of them are not.

k-way Independence

Events $A_1, A_2, \ldots$ are

k-way independent

iff any k of them are mutually independent.

Pairwise = 2-way

k-way Independence

Events $A_1, A_2, \ldots$ are

k-way independent

iff any k of them are mutually independent.

$O, H_1, \ldots, H_k$ are k-way, not (k+1)-way independent

Mutual Independence

Events $A_1, A_2, \ldots, A_n$ are

mutually independent

when they are n-way independent

$$\left(\begin{array}{l} 2^n-(n+1) \text{ equations} \\ \text{to check!} \end{array}\right)$$