Mathematics for Computer Science
MIT 6.042J/18.062J

Expected Time to Failure

Mean Time to "Failure"

Flip a coin until a Head comes up

Pr[Head] = p

F ::= #flips to 1st Head

E[F] ?

Mean Time to "Failure"

Pr[F= 1] = Pr[H] = p

Mean Time to "Failure"

Pr[F= 1] = Pr[H] = p

Pr[F= 2] = Pr[TH] = $q \cdot p$

Pr[F= 3] = Pr[TTH] = $q^2 \cdot p$

$$\text{PDF}_F(n) = q^{n-1}p$$

Geometric Distribution

Mean Time to "Failure"

$$E[F] = \Sigma_{n>0}\, n \cdot Pr[F=n]$$
$$= \Sigma_{n>0}\, n \cdot q^{n-1} p$$
$$= p \underbrace{\Sigma_{n \geq 0}\, (n+1) q^n}_{\dfrac{1}{(1-q)^2}}$$

Mean Time to "Failure"

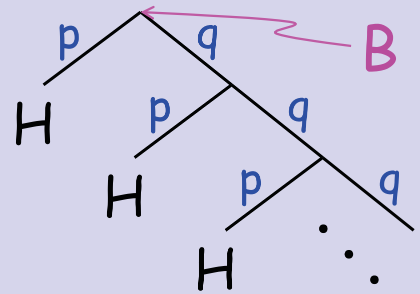
$$E[F] = \Sigma_{n>0}\, n \cdot Pr[F=n]$$
$$= \Sigma_{n>0}\, n \cdot q^{n-1} p$$
$$= p \frac{1}{(1-q)^2}$$

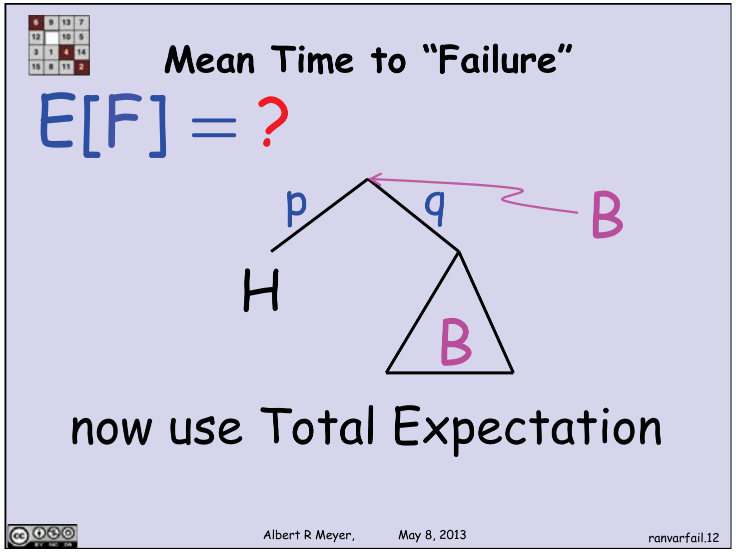
Mean Time to "Failure"

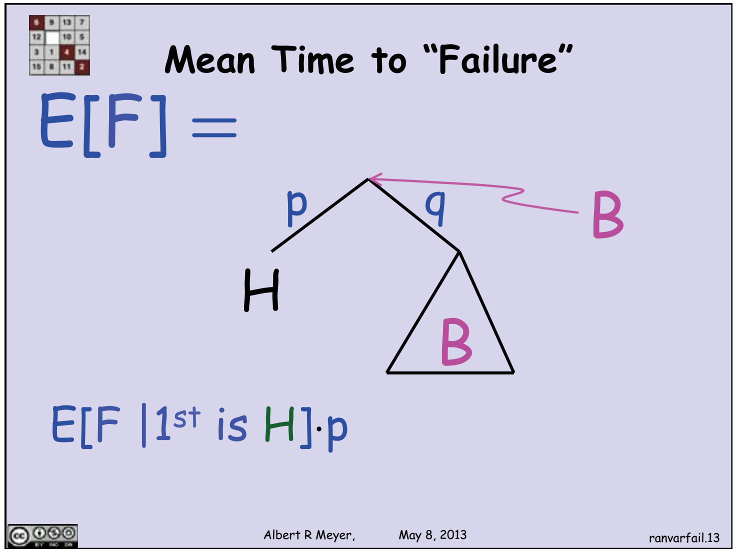
$$E[F] = \Sigma_{n>0}\, n \cdot Pr[F=n]$$
$$= \Sigma_{n>0}\, n \cdot q^{n-1} p$$
$$= p \frac{1}{p^2} = \frac{1}{p}$$

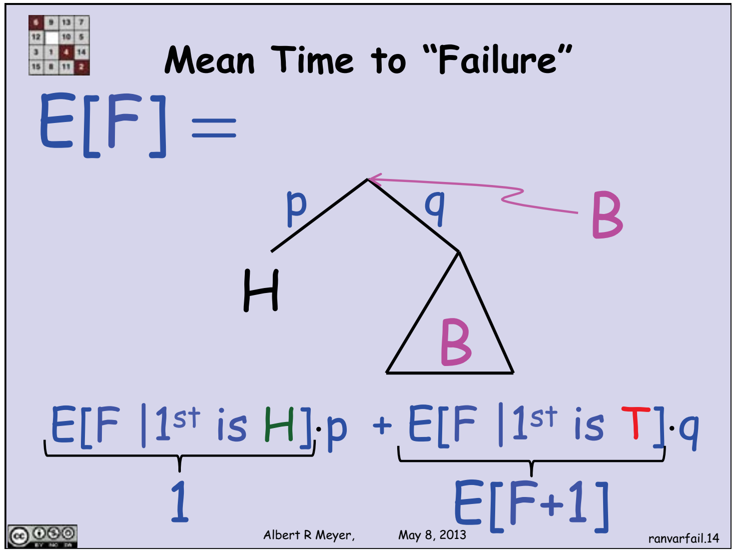
Mean Time to "Failure"

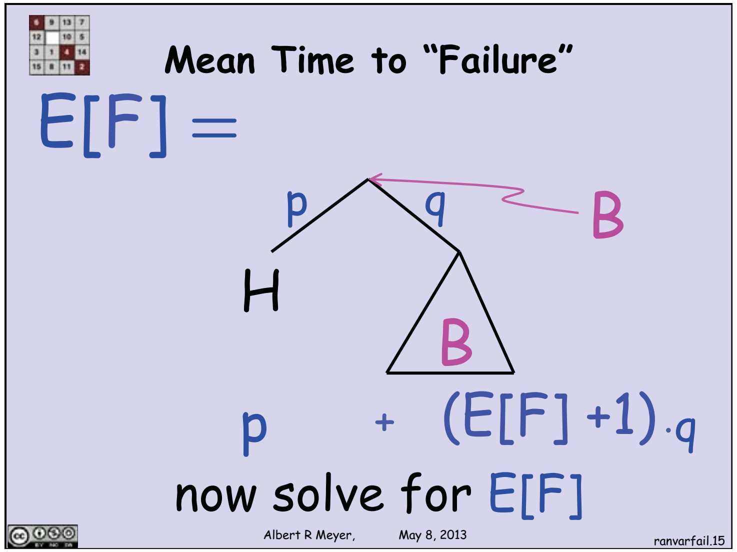
$$E[F] = ?$$




Mean Time to "Failure"
E[F] = ?
p
q
B
H
B
now use Total Expectation
Albert R Meyer, May 8, 2013
ranvarfail.12


Mean Time to "Failure"
E[F] =
p
q
B
H
B
E[F |1st is H]·p
Albert R Meyer, May 8, 2013
ranvarfail.13


Mean Time to "Failure"
E[F] =
p
q
B
H
B
E[F |1st is H]·p + E[F |1st is T]·q
1
E[F+1]
Albert R Meyer, May 8, 2013
ranvarfail.14


Mean Time to "Failure"
E[F] =
p
q
B
H
B
p + (E[F] +1)·q
now solve for E[F]
Albert R Meyer, May 8, 2013
ranvarfail.15

Mean Time to "Failure"

$$E[F] = \frac{1}{p}$$

Mean Time to Failure

application: if space station Mir has 1/150,000 chance of destruction in any given hour, how may hours expected until destruction?

150,000 hours $\approx$ 17 years

Intuitive argument

$$E[\#\text{fails in 1 try}] = p$$

$$E[\#\text{fails in n tries}] = np$$

$$E[\#\text{tries between fails}]$$

$$= \frac{\#\text{tries}}{\#\text{fails}} = \frac{n}{np} = \frac{1}{p}$$

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