



Mathematics for Computer Science
MIT 6.042J/18.062J

Expected Time to Failure



Albert R Meyer, May 8, 2013 ranvarfail.1



Mean Time to "Failure"

Flip a coin until a Head comes up
 $\Pr[\text{Head}] = p$
 $F ::= \# \text{flips to 1st Head}$
 $E[F] ?$



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Mean Time to "Failure"

$$\Pr[F=1] = \Pr[H] = p$$


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Mean Time to "Failure"

$$\Pr[F=1] = \Pr[H] = p$$

$$\Pr[F=2] = \Pr[TH] = q \cdot p$$

$$\Pr[F=3] = \Pr[TTH] = q^2 \cdot p$$

$$\text{PDF}_F(n) = q^{n-1} p$$

Geometric Distribution



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Mean Time to "Failure"

$$\begin{aligned}
 E[F] &= \sum_{n>0} n \cdot \Pr[F=n] \\
 &= \sum_{n>0} n \cdot q^{n-1} p \\
 &= p \underbrace{\sum_{n \geq 0} (n+1) q^n}_{\frac{1}{(1-q)^2}}
 \end{aligned}$$

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Mean Time to "Failure"

$$\begin{aligned}
 E[F] &= \sum_{n>0} n \cdot \Pr[F=n] \\
 &= \sum_{n>0} n \cdot q^{n-1} p \\
 &= p \frac{1}{(1-q)^2}
 \end{aligned}$$

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Mean Time to "Failure"

$$\begin{aligned}
 E[F] &= \sum_{n>0} n \cdot \Pr[F=n] \\
 &= \sum_{n>0} n \cdot q^{n-1} p \\
 &= p \frac{1}{p^2} = \frac{1}{p}
 \end{aligned}$$

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Mean Time to "Failure"

$E[F] = ?$

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Mean Time to "Failure"

$E[F] = ?$

now use Total Expectation

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Mean Time to "Failure"

$E[F] =$

$E[F | 1^{st} \text{ is } H] \cdot p$

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Mean Time to "Failure"

$E[F] =$

$E[F | 1^{st} \text{ is } H] \cdot p + E[F | 1^{st} \text{ is } T] \cdot q$

$1 \quad E[F+1]$

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Mean Time to "Failure"

$E[F] =$

$p + (E[F] + 1) \cdot q$

now solve for $E[F]$

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Mean Time to "Failure"

$$E[F] = \frac{1}{p}$$


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Mean Time to Failure

application: if space station Mir has $1/150,000$ chance of destruction in any given hour, how many hours expected until destruction?

$150,000$ hours \approx 17 years



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Intuitive argument

$$E[\text{\#fails in 1 try}] = p$$

$$E[\text{\#fails in } n \text{ tries}] = np$$

$$E[\text{\#tries between fails}] = \frac{\text{\# tries}}{\text{\# fails}} = \frac{n}{np} = \frac{1}{p}$$


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