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|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

• *Mathematics for Computer Science*
 MIT 6.042J/18.062J

Stable Matching: The Mating Ritual (day by day)



Albert R Meyer. April 3, 2013

stable-ritual.1

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|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual

Morning: boy serenades favorite girl



Billy Bob

Brad

Angelina



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| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual

Morning: boy serenades favorite girl
Afternoon: girl **rejects** all but favorite

If you're not Brad
 take a hike!



Billy Bob

Brad

Angelina



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|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual

- **Morning:** boy serenades **his** favorite girl
- **Afternoon:** girl rejects all but **her** favorite boy
- **Evening:** rejected boy writes off girl



Billy Bob



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| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual

Stop when no girl rejects.
Each girl marries her favorite suitor (if any).



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|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Stable Marriages

Termination:

There exists a Wedding Day.

Partial Correctness:

Everyone is married.

Marriages are stable.



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|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual: **Termination**

total # names remaining
on boys' lists:

strictly decreasing
& \mathbb{N} -valued

So \exists Wedding Day



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|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual: **Correctness**



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|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual: **girls improve**

Lemma:

A girl's favorite tomorrow
will be at least as desirable
to her as today's.
...because today's favorite will
stay until she rejects him for
someone better.



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stable-ritual.10

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual: **girls improve**

Lemma:

The rank of a girl's
favorite is **weakly
increasing**



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| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual: **boys get worse**

Lemma:

A boy's favorite tomorrow
will be no more desirable
to him than today's.
...because boys work
straight down their lists.



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| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual: **boys get worse**

Lemma:

The rank of girl a boy
serenades is **weakly
decreasing**



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| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual: **invariant**

If G is not on B 's list, then she has a better current favorite.

Proof: When G rejected B she had a better suitor (her favorite that day), and her favorites never get worse.



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| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

On Wedding Day

Each girl has ≤ 1 suitor.
(by def of wedding day)
Each boy is married, or has no girls on his list.



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| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
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| 3 | 1 | 4 | 14 |
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Mating Ritual: **No bigamy**

No husband sharing ...because boys serenade
one girl at a time.



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| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
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| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual: **Everyone marries**

Everyone is married on wedding day

Proof: By **contradiction**.

If B is not married, his list is empty. By **invariant**, all girls have favorites better than B – so they do have a favorite: all **girls** are married. Since no bigamy, all **boys** are married.



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| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual: **Stable marriages**

Marriages are Stable:

Bob won't be in rogue couple with
case 1: a girl G on his final list,
 since he's already married to the
 best of them.



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| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual: **Stable marriages**

Marriages are Stable:

Bob won't be in rogue couple with
case 2: a girl G not on his list,
 since by invariant, G likes her
 spouse better than Bob.



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| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mating Ritual: **Stable marriages**

Marriages are Stable:

Bob is not in a rogue couple
 There are **no rogue couples**



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6.042J / 18.062J Mathematics for Computer Science
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