Mathematics for Computer Science

MIT 6.042J/18.062J

Bounds on Deviation
Markov Bound

Example: IQ

IQ measure was constructed so that

average IQ = 100.

What fraction of the people can possibly have an IQ ≥ 300?

...at most 1/3

IQ Higher than 300?

If more than 1/3 have IQ ≥ 300, then

avg $> (1/3) \cdot 300 = 100$!

—a contradiction

IQ Higher than 300?

At most 1/3 of people have IQ ≥ 300

$$\Pr[\text{IQ} \geq 300] \leq \frac{E[\text{IQ}]}{300}$$

IQ Higher than x?

In general,

$$\Pr[IQ \geq x] \leq \frac{100}{x}$$

IQ Higher than x?

Besides mean = 100,
we used only one fact about the distribution of IQ:
IQ is always nonnegative

Markov Bound

If R is nonnegative, then

$$\Pr[R \geq x] \leq \frac{E[R]}{x}$$

for $x > E[R]$

Markov Bound (Restated)

Let $x = c \cdot E[R]$:

$$\Pr[R \geq c\mu] \leq \frac{1}{c}$$

$\Pr[R \geq 3 \cdot \text{expected}] \leq 1/3$

Markov Bound

- Weak
- Obvious
- Useful anyway

IQ ≥ 300, again

Suppose we are given that IQ is always ≥ 50?

Get a better bound using

$(IQ - 50)$

since this is now ≥ 0.

IQ ≥ 300, again

$$\Pr[IQ \geq 300] =$$

$$\Pr[IQ - 50 \geq 300 - 50]$$

$$\leq \frac{100 - 50}{300 - 50} = \frac{1}{5}$$

Improved Markov Bound

Better bound from Markov by shifting R to have 0 as minimum

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