

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Independent Events



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Independent Events

Definition 1:

Events A and B are independent iff

$$\Pr[A] = \Pr[A \mid B]$$

Definition 2:

Events A and B are independent iff

$$\Pr[A] \cdot \Pr[B] = \Pr[A \cap B]$$



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Definitions of Independence

proof of equivalence:

$$\Pr[A] = \Pr[A \mid B] \quad \text{iff}$$

$$\Pr[A] = \frac{\Pr[A \cap B]}{\Pr[B]} \quad \text{iff}$$

$$\Pr[A] \cdot \Pr[B] = \Pr[A \cap B]$$



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Definitions of Independence

need $\Pr[B] \neq 0$ for Def. 1.

Def. 2 always works:

$$\Pr[A] \cdot \Pr[B] = \Pr[A \cap B]$$



Independence

$$\Pr[A] \cdot \Pr[B] = \Pr[A \cap B]$$

symmetric in A and B so,
 A independent of B iff
 B independent of A



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indep-events.5

Independence

Corollary: If $\Pr[B] = 0$, then
 B is independent of every
event



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indep-events.6

Independence

Corollary: If $\Pr[B] = 0$, then
 B is independent of every
event - even itself.



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Independence

A independent of B
means



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indep-events.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Independence

A independent of B
means A is independent of
whether or not B occurs:



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indep-events.10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Independence

Lemma:

A independent of B iff
 A independent of \bar{B} .



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indep-events.11

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Independence

Lemma:

A independent of B iff
 A independent of \bar{B}

Simple proof using:

$$\Pr[A - B] = \Pr[A] - \Pr[A \cap B]$$



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indep-events.12

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