Mathematics for Computer Science

MIT 6.042J/18.062J

Equivalence Relations

two-way walks

walk from u to v and back from v to u:
u and v are strongly connected.

$u\ G^*\ v$ AND $v\ G^*\ u$

symmetry

relation R on set A is symmetric iff

a R b IMPLIES b R a

equivalence relations

transitive,
symmetric &
reflexive

equivalence relations

Theorem:

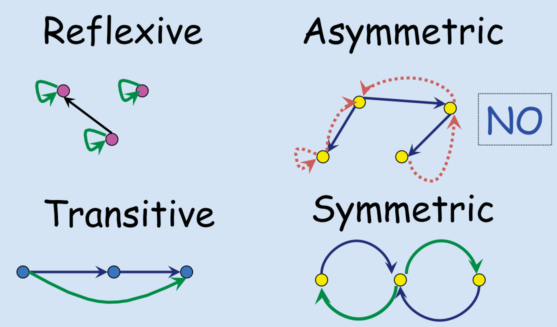
R is an equiv rel iff
R is the strongly connected relation of some digraph

equivalence relations

examples:

- $=$ (equality)
- $\equiv$ (mod n)
- same size
- same color

Graphical Properties of Relations



Representing Equivalences

Representing equivalences

For total function $f:A \rightarrow B$

define relation $\equiv_f$ on A:

$a \equiv_f a'$ IFF $f(a) = f(a')$

Representing equivalences

Theorem:

Relation R on set A is an equiv. relation IFF

R is $\equiv_f$

for some $f:A \rightarrow B$

representing $\equiv \pmod{n}$

$\equiv \pmod{n}$ is

$\equiv_f$ where

$f(k) ::= \text{rem}(k,n)$

Representing equivalences

For partition Π of A

define relation $\equiv_\Pi$ on A:

$a \equiv_\Pi a'$ IFF a, a' are in the same block of Π

Representing equivalences

Theorem:

Relation R on set A is an equiv. relation IFF

$$R \text{ is } \equiv_{\Pi}$$

for some partition Π of A