

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Equivalence Relations

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equiv.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

two-way walks

walk from u to v and
back from v to u :
 u and v are **strongly
connected**.

$$u G^* v \text{ AND } v G^* u$$

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equiv.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

symmetry

relation R on set A
is **symmetric** iff

$$a R b \text{ IMPLIES } b R a$$

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equiv.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

equivalence relations

transitive,
symmetric &
reflexive

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equiv.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

equivalence relations

Theorem:

R is an **equiv rel** iff
 R is the **strongly connected** relation
of some digraph

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

equivalence relations

examples:

- = (equality)
- \equiv (mod n)
- same size
- same color

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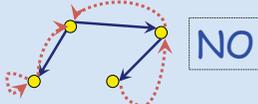
6	9	13	7
12		10	5
3	1	4	14
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Graphical Properties of Relations

Reflexive



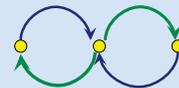
Asymmetric



Transitive



Symmetric



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6	9	13	7
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Representing Equivalences

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6	9	13	7
12		10	5
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Representing equivalences

For total function $f:A \rightarrow B$
 define relation \equiv_f on A :
 $a \equiv_f a'$ IFF $f(a) = f(a')$



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equiv.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Representing equivalences

Theorem:

Relation R on set A is
 an equiv. relation IFF

R is \equiv_f
 for some $f:A \rightarrow B$



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equiv.10

6	9	13	7
12		10	5
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15	8	11	2

representing $\equiv \pmod n$

$\equiv \pmod n$ is

\equiv_f where

$f(k) ::= \text{rem}(k,n)$



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equiv.11

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Representing equivalences

For partition Π of A
 define relation \equiv_Π on A :
 $a \equiv_\Pi a'$ IFF a, a' are in
 the same block of Π



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equiv.12

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Representing equivalences

Theorem:

Relation R on set A is an
equiv. relation IFF

$$R \text{ is } \equiv_{\Pi}$$

for some partition Π of A



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