

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

# Deviation from the Mean



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Don't expect the Expectation!

Toss 101 fair coins.

$$E[\#Heads] = 50.5$$

$$\Pr[\text{exactly } 50.5 \text{ Heads}] = 0$$

$$\Pr[\text{exactly } 50 \text{ Heads}] < 1/13$$

$$\Pr[50.5 \pm 1 \text{ Heads}] < 1/7$$



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Don't expect the Expectation!

Toss 1001 fair coins.

$$E[\#Heads] = 500.5$$

$$\Pr[\#H = 500] < \underbrace{1/39}$$

$$\Pr[\#H = 500.5 \pm 1] < \underbrace{1/19}_{\text{smaller}}$$



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Don't expect the Expectation!

As #tosses grows,  
#Heads gets less likely  
to be within a fixed  
distance of the mean




**Within a % of the mean?**  
 Toss 1001 fair coins. of 1001  
 $\Pr[\#H = 500.5 \pm 1\%]$   
 $= \Pr[\#H = 500.5 \pm 10]$   
 $\approx 0.49$   
 not so bad


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**Giving Meaning to the Mean**  
 Let  $\mu ::= E[R]$ . What is  
 $\Pr[R \text{ far from } \mu]$ ?  
 $\Pr[|R - \mu| > x]$   
 R's average deviation?  
 $E[|R - \mu|]$ ?


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**Two Dice with Same Mean**

**Fair Die**

- $E[D_1] = 3.5$

**Loaded Die throwing only 1 & 6:**

- $E[D_2] = (1+6)/2 = 3.5$  also!




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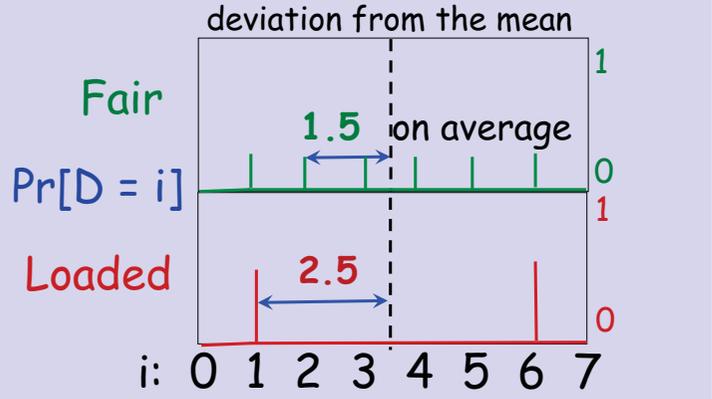

**Two Dice with Same Mean**

deviation from the mean

**Fair**

$\Pr[D = i]$

**Loaded**




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## Dice have Different Deviations

Fair Die:

$$E[ |D_1 - \mu| ] = 1.5$$

Loaded Die:

$$E[ |D_2 - \mu| ] = 2.5$$



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May 10, 2013

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## Giving Meaning to the Mean

The mean alone is not a good predictor of **R**'s behavior.

We generally need more about its distribution, especially probable deviation from its mean.



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May 10, 2013

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