Mathematics for Computer Science

MIT 6.042J/18.062J

Deviation from the Mean

Don't expect the Expectation!

Toss 101 fair coins.

E[#Heads] = 50.5

Pr[exactly 50.5 Heads] = 0

Pr[exactly 50 Heads] < 1/13

Pr[50.5 $\pm$ 1 Heads] < 1/7

Don't expect the Expectation!

Toss 1001 fair coins.

E[#Heads] = 500.5

Pr[#H = 500] < 1/39

Pr[#H = 500.5 $\pm$ 1] < 1/19

smaller

Don't expect the Expectation!

As #tosses grows, #Heads gets less likely to be within a fixed distance of the mean

Within a % of the mean?

Toss 1001 fair coins.

Pr[#H = 500.5 ± 1% of 1001]

= Pr[#H = 500.5 ± 10]

≈ 0.49

not so bad

Giving Meaning to the Mean

Let $\mu ::= E[R]$. What is

Pr[R far from μ]?

$Pr[\,|R-\mu|>x\,]$

R's average deviation ?

$E[\,|R-\mu|\,]$?

Two Dice with Same Mean

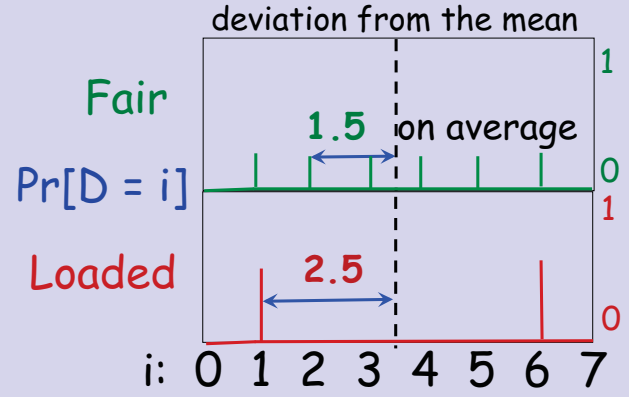
Fair Die

- $E[D_1] = 3.5$

Loaded Die throwing only 1 & 6:

- $E[D_2] = (1+6)/2 = 3.5$ also!

Two Dice with Same Mean



Dice have Different Deviations

Fair Die:

$E[\,|D_1 - \mu|\,] = 1.5$

Loaded Die:

$E[\,|D_2 - \mu|\,] = 2.5$

Giving Meaning to the Mean

The mean alone is not a good predictor of R's behavior.

We generally need more about its distribution, especially probable deviation from its mean.

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