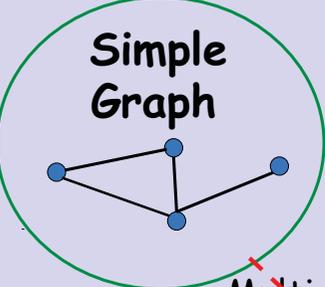



Mathematics for Computer Science
 MIT 6.042J/18.062J

Simple Graphs: Degrees

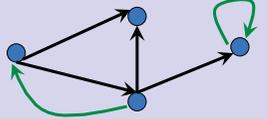

 Albert R Meyer April 1, 2013 degrees.1


Types of Graphs



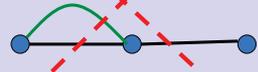
Simple Graph

this week



Directed Graph

last week



~~Multi-Graph~~


 Albert R Meyer April 1, 2013 degrees.2


A simple graph:

Definition:

A simple graph G consists of

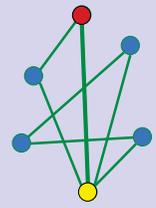
- a nonempty set, V , of vertices, and
- a set, E , of edges such that each edge has two endpoints in V


 Albert R Meyer April 1, 2013 degrees.3


A Simple Graph

vertices, V

undirected edges, E



edge



$::= \{ \bullet, \bullet \}$

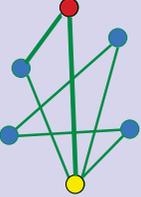
"adjacent"


 Albert R Meyer April 1, 2013 degrees.4

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Vertex degree

degree of a vertex is
of **incident** edges



$\text{deg}(\bullet) = 2$



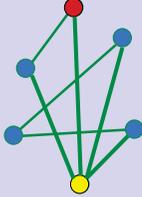
Albert R Meyer April 1, 2013

degrees.6

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Vertex degree

degree of a vertex is
of **incident** edges



$\text{deg}(\bullet) = 4$



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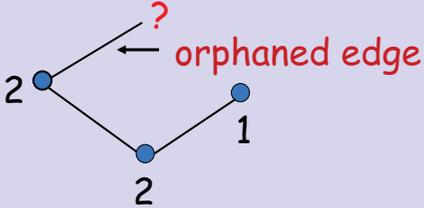
degrees.7

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Impossible Graph

Is there a graph with
vertex degrees 2,2,1?

NO!



orphaned edge



Albert R Meyer April 1, 2013

degrees.8

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Handshaking Lemma

sum of degrees is
twice # edges

$$2|E| = \sum_{v \in V} \text{deg}(v)$$

Proof: Each edge contributes
2 to the sum on the right



Albert R Meyer April 1, 2013

degrees.9

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Handshaking Lemma

sum of degrees is
twice # edges

$$2|E| = \sum_{v \in V} \deg(v)$$

2+2+1 = odd,
so impossible

 Albert R Meyer April 1, 2013 degrees.10

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Sex in America: Men more Promiscuous?

- Univ Chicago, *The Social Organization of Sexuality*, 1994: men have 74% more women partners than women have men
- NBC News, *American Sex Survey*, 2004: men have 233% more women partners
- U.S. National Center for Health Statistics study, 2007: men have 175% more women partners

 Albert R Meyer April 1, 2013 sexgraph.11

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Sex in America: Men more Promiscuous?

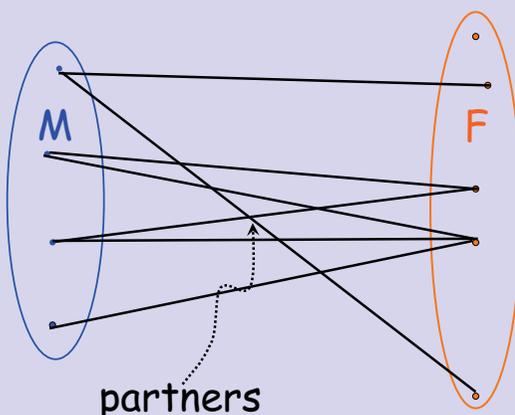
Studies claim different %'s
but agree that
men average many more
partners than women.

Graph theory shows
this is nonsense

 Albert R Meyer April 1, 2013 sexgraph.12

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Sex Partner Graph



 Albert R Meyer April 1, 2013 degrees.13

Counting pairs of partners

$$\text{avg degree}(M) ::= \frac{\sum_{m \in M} \text{deg}(m)}{|M|}$$

$$\text{avg degree}(F) ::= \frac{\sum_{f \in F} \text{deg}(f)}{|F|}$$

Albert R Meyer April 1, 2013 degrees.14

Counting pairs of partners

$$\frac{\text{avg degree}(M)}{\text{avg degree}(F)} = \frac{\sum_{m \in M} \text{deg}(m)}{\sum_{f \in F} \text{deg}(f)}$$

$$\sum_{m \in M} \text{deg}(m) = |E| = \sum_{f \in F} \text{deg}(f)$$

Albert R Meyer April 1, 2013 degrees.16

Counting pairs of partners

$$\frac{\text{avg degree}(M)}{\text{avg degree}(F)} = \frac{|F|}{|M|}$$

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Average number of partners

$$\text{avg-deg}(M) = 1.035 \cdot \text{avg-deg}(F)$$

Averages differ solely by
ratio of females to males.
No big difference
Nothing to do with promiscuity

Albert R Meyer April 1, 2013 degrees.19

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Why are surveys wrong?

Maybe people are **lying**:

- Males exaggerate?
- Females deny?

Maybe Males have partners
outside the study



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