



Mathematics for Computer Science  
MIT 6.042J/18.062J

# Matching Birthdays

Albert R Meyer, December 1, 2013 birthday.1



## Birthday Pairs

$P ::=$  # pairs with matching b'days among  $n$  people in a  $d$ -day year

$$P = \sum_{1 \leq i < j \leq n} M_{ij}$$

$M_{ij} ::=$  indicator that  $i^{\text{th}}$  &  $j^{\text{th}}$  birthdays match

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## Birthday Pairs

$E[M_{ij}] = 1/d$   
so by linearity of  $E[\ ]$

$$E[P] = \sum_{1 \leq i < j \leq n} E[M_{ij}] = \binom{n}{2} \cdot \frac{1}{d}$$

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## Birthday Pairs

Have data on 179\* students

$$E[P] = \binom{179}{2} \cdot \frac{1}{365} \approx 43.6$$

\*excluding 2 sets of twins

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Birthday Pairs

How likely is  $P$  near 43.6?

$$\Pr[|P - 43.6| > k]$$

hard to calculate!

Variance easy to calculate!



Albert R Meyer,

December 1, 2013

birthday.6

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Pairwise Independence

[Albert and Drew have same b'day]

is independent of

[David and Mike have same b'day]

that is,  $M_{\text{Albert,Drew}}$  &  $M_{\text{David, Mike}}$

are independent

Obvious since the b'days of Albert, Drew, David & Mike are mutually independent



Albert R Meyer,

December 1, 2013

birthday.7

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Pairwise Independence

[Albert and Drew have same b'day]

is independent of

[Albert and Mike have same b'day]

that is,  $M_{\text{Albert,Drew}}$  &  $M_{\text{Albert, Mike}}$

are independent

(pairwise, but not 3-way:

$M_{\text{Drew, Mike}}$  depends on other two)



Albert R Meyer,

December 1, 2013

birthday.8

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Birthday Pairs

$$\text{Var}[M_{ij}] = (1/365)(1 - 1/365)$$

so by prwise additivity of  $\text{Var}[\ ]$

$$\text{Var}[P] = \sum \text{Var}[M_{ij}] = \binom{179}{2} \text{Var}[M_{ij}]$$

$$= \binom{179}{2} \cdot \frac{1}{365} \cdot \left(1 - \frac{1}{365}\right) \approx 43.5$$

$$\sigma_p < 6.6$$



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December 1, 2013

birthday.9



### Birthday Predictions

Chebyshev:

$$\Pr[43.6 \pm 2\sigma \text{ pairs}] > 1 - (1/2)^2 = 3/4$$

32 to 55 pairs 75% of the time

We actually found 47 pairs  
(29 pairs & 6 triples)



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### Fall '13 Matching Birthdays

179 Fall '13 students: 29 Pairs & 6 Triples

2/9	4/20	7/29	9/25
2/23	5/11	8/7	10/2
3/7	6/8	8/9	10/23
3/24	6/9	8/22	10/25
3/26	6/15	8/23	10/30
3/28	7/7	8/29	11/1
3/29	7/14	9/5	11/8
4/13	7/15	9/17	12/4
4/19	7/27	9/21	



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6.042J / 18.062J Mathematics for Computer Science  
Spring 2015

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