

Mathematics for Computer Science  
MIT 6.042J/18.062J

# Introduction to Random Variables Bigger Number Game



Albert R Meyer May 6, 2013

ranvarbigger.1

## Guess the Bigger Number

### Team 1:

- Write two integers from 0 to 7 on two pieces of paper
- Show to Team 2 face down

### Team 2:

- Expose one paper and look at number
- Either *stick* or *switch* to other number

Team 2 wins if gets larger number



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## Guess the Bigger Number

Do you think one team has an advantage?



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## Guess the Bigger Number

Do you think one team has an advantage? Which one?

You might like to try playing the game a few times with some teammates before seeing the answers below.



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Strategy for Team 2

- pick a paper to expose, giving each paper equal probability.
- if exposed number is "small" then switch, otherwise stick. That is switch if  $\leq$  threshold  $Z$  where  $Z$  is a random integer  $\in [0,7)$



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Analysis of Team 2 Strategy

Let  $low < high$  be the integers chosen by Team 1. There are three cases:



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Analysis of Team 2 Strategy

Case  $M$ :  $low \leq Z < high$   
 Team 2 wins in this case, so  
 $Pr[\text{Team 2 wins} \mid M] = 1$   
 and  $Pr[M] \geq \frac{1}{7}$



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Analysis of Team 2 Strategy

Case  $H$ :  $high \leq Z$   
 Team 2 will switch, so wins iff  
 $low$  card gets exposed  
 $Pr[\text{Team 2 wins} \mid H] = \frac{1}{2}$



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### Analysis of Team 2 Strategy

Case L:  $Z < \text{low}$

Team 2 will stick, so wins iff high card gets exposed

$$\Pr[\text{Team 2 wins} \mid L] = \frac{1}{2}$$


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### Analysis of Team 2 Strategy

So  $\geq 1/7$  of time, sure win.

Rest of time, win  $1/2$ .

By Law of Total Probability



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### Analysis of Team 2 Strategy

So  $\geq 1/7$  of time, sure win.

Rest of time, win  $1/2$ .

$$\Pr[\text{Team 2 wins}] = \Pr[\text{win} \mid M] \cdot \Pr[M] + \Pr[\text{win} \mid \bar{M}] \cdot \Pr[\bar{M}]$$


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### Analysis of Team 2 Strategy

So  $\geq 1/7$  of time, sure win.

Rest of time, win  $1/2$ .

$$\Pr[\text{Team 2 wins}] \geq 1 \cdot \frac{1}{7} + \frac{1}{2} \cdot \left(1 - \frac{1}{7}\right) = \frac{4}{7}$$


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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Analysis of Team 2 Strategy

So Team 2 has the advantage



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6	9	13	7
12	10	5	
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15	8	11	2

## Analysis of Team 2 Strategy

So Team 2 has the advantage, no matter what Team 1 does!



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6	9	13	7
12	10	5	
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15	8	11	2

## Team 1 Strategy

...& Team 1 can play so  
 $\Pr[\text{Team 2 wins}] \leq \frac{4}{7}$   
 no matter what



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Optimal Strategy

$\Pr[\text{Team 2 wins}] = \frac{4}{7}$   
 is optimal for both



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Random Variables

Informally: an **RV** is a number produced by a **random process**:

- threshold variable **Z**
- number of exposed card
- number of larger card
- number of smaller card



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ranvarindep.19

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