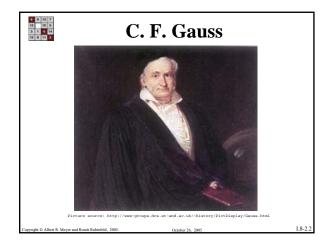


Sums & Products

October 26, 2005 L8-2.1



* (2) (3) (2) (2) (3) (3) (3) (4) (4) (5) (5) (4) (2)

Sum for Children

 $414 + \cdots + 453 + 466$

8 W 13 73 12 10 S 3 1 4 54 15 D 11 2

Sum for Children

Nine-year old Gauss saw
30 numbers each 13 greater
than the previous one.

(So the story goes.)

L8-2.



Sum for Children

$$1^{st} + 30^{th} = 89 + 466$$
 = 555
 $2^{nd} + 29^{th} =$ (1st+13) + (30th-13) = 555
 $3^{rd} + 28^{th} =$ (2nd+13) + (29th-13) = 555



Sum for Children

Sum of k^{th} term and $(31-k)^{th}$ term is invariant!

Total =
$$555 \cdot 15$$

= $(1^{st} + last) \cdot (\# terms/2)$
= $((1^{st} + last)/2) \cdot (\# terms)$

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Sum for Children

Example:

$$1 + 2 + \dots + (n-1) + n = \frac{(1+n)n}{2}$$



Geometric Series

$$G_n := 1 + x + x^2 + \dots + x^{n-1} + x^n$$

 $xG_n = x + x^2 + x^3 + \dots + x^n + x^{n+1}$

Geometric Series

$$G_{n} ::= 1 + x + x^{2} + \dots + x^{n-1} + x^{n}$$

$$xG_{n} = x + x^{2} + x^{3} + \dots + x^{n} + x^{n+1}$$

$$G_{n} - xG_{n} = 1$$

$$- x^{n+1}$$

Geometric Series

$$G_n := 1 + x + x^2 + \dots + x^{n-1} + x^n$$

$$xG_n = x + x^2 + x^3 + \dots + x^n + x^{n+1}$$

$$G_n - xG_n = 1 - x^{n+1}$$

$$G_n - xG_n = 1 - x^{n+1}$$

$$G_n = \frac{1 - x^{n+1}}{1 - x}$$

9 13 7. 12 18 5 3 1 4 54 15 9 11 2

Geometric Series

$$G_n = \frac{1 - x^{n+1}}{1 - x}$$

Consider the *infinite* sum (series)

$$1 + x + x^{2} + \dots + x^{n-1} + x^{n} + \dots + = \sum_{i=0}^{\infty} x^{i}$$

Infinite Geometric Series

$$G_n = \frac{1 - x^{n+1}}{1 - x}$$

$$\lim_{n \to \infty} G_n = \frac{1 - \lim_{n \to \infty} x^{n+1}}{1 - x} = \frac{1}{1 - x}$$



Infinite Geometric Series

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

for x < 1



Team Problem

Problem 1



The future value of \$\$

I will promise to pay you \$100 in exactly one year, if you will pay me \$X now.



The future value of \$\$

My bank will pay me 3% interest.

Define *bankrate*:

$$b := 1.03$$

-- the factor by which bank will increase my holdings in 1 year.



The future value of \$\$

If I deposit your \$X for a year, I will have $(b\cdot X)$.

So I won't lose money as long as

 $bX \ge 100$.

 $X \ge \$100/1.03 \approx \97.09



The future value of \$\$

- \$1 in a year is worth \$ 0.9709 today
- n is worth n a year earlier,

where r := 1/b.

• So \$*n* paid in two years is worth \$nr paid in one year, and is worth nr^2 today.



The future value of \$\$

n paid k years from now is worth nr^k today where r := 1/b and nr^k today

minha O. A Bross D. Marrow and Domin Dubinful A 2006



Annuities

I will pay you \$100/year for 10 years if you will pay me \$Y now.

I can't lose if you pay me

$$100r + 100r^2 + 100r^3 + \dots + 100r^{10}$$

$$=100r(1+r+...+r^9)$$

$$= 100r(1-r^{10})/(1-r) = $853.02$$

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Annuities

I will pay you \$100/year for 10 years if you will pay me \$853.02 now.

QUICKIE: If bankrates unexpectedly increase in the next few years,

- A. I come out ahead
- B. You come out ahead
- C. The deal stays fair

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1



Manipulating Sums

$$\frac{d}{dx} \qquad \sum_{i=0}^{n} x^{i} = \frac{1 - x^{n+1}}{1 - x}$$

$$\sum_{i=0}^{n} ix^{i-1} = \frac{1}{x} \sum_{i=1}^{n} ix^{i} = \frac{d}{dx} \left(\frac{1 - x^{n+1}}{1 - x} \right)$$

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tober 26, 2005

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Manipulating Sums

$$\sum_{i=1}^{n} ix^{i} = \frac{x - (n+1)x^{n+1} + nx^{n+2}}{(1-x)^{2}}$$

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Team Problem

Problems 2,3

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L8-2.24