Mathematics for Computer Science

MIT 6.042J/18.062J

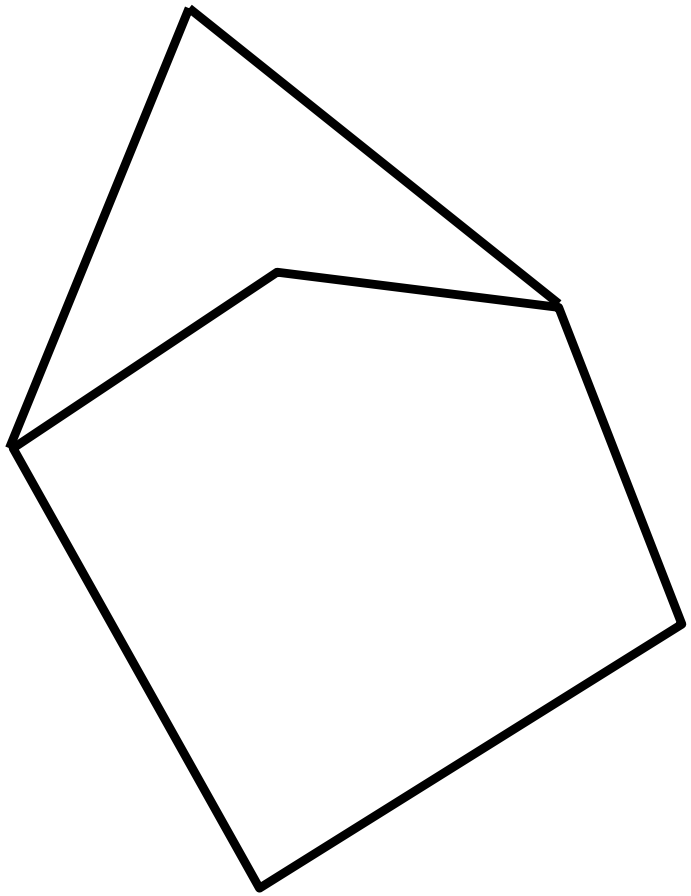
Drawing Planar Graphs

Planar Graphs

A graph is ***planar*** if there is a way to draw it in the plane without edges crossing.

Planar Graphs

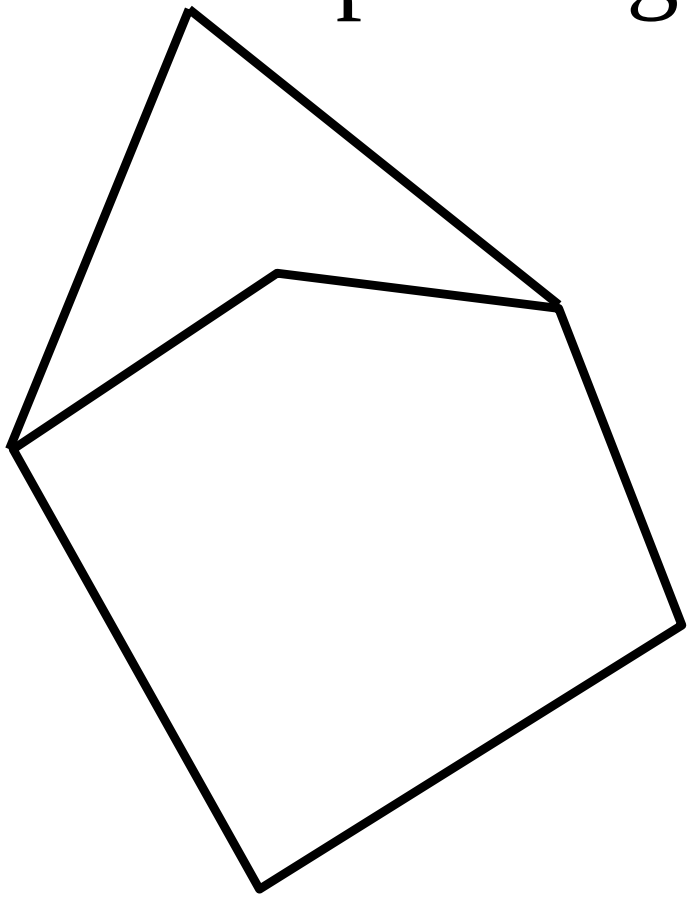
Maps are 2-connected planar graphs



Planar Graphs

Maps are 2-connected planar graphs

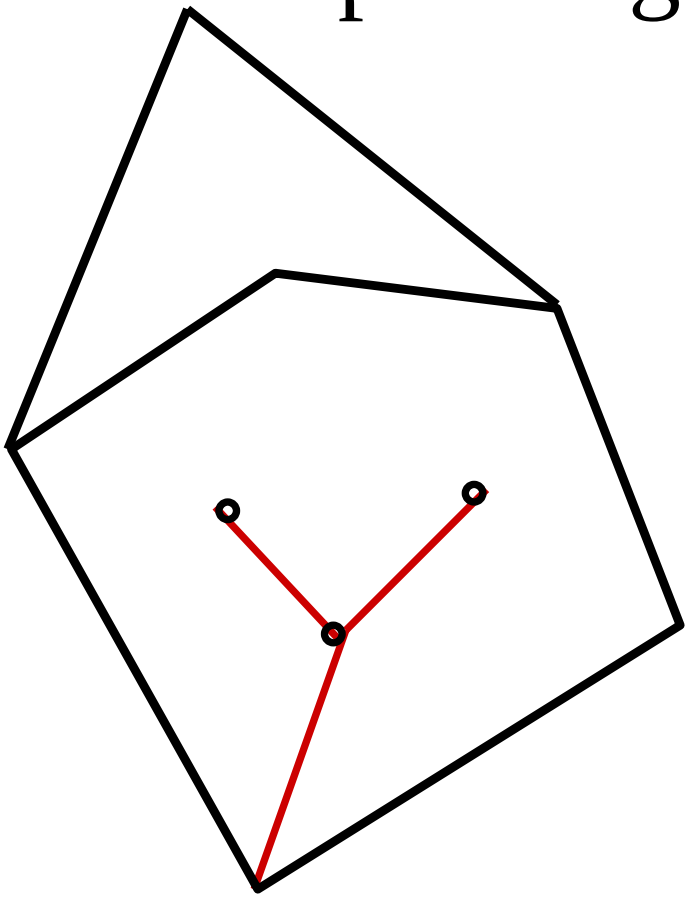
General connected planar graphs may have



Planar Graphs

Maps are 2-connected planar graphs

General connected planar graphs may have dongles



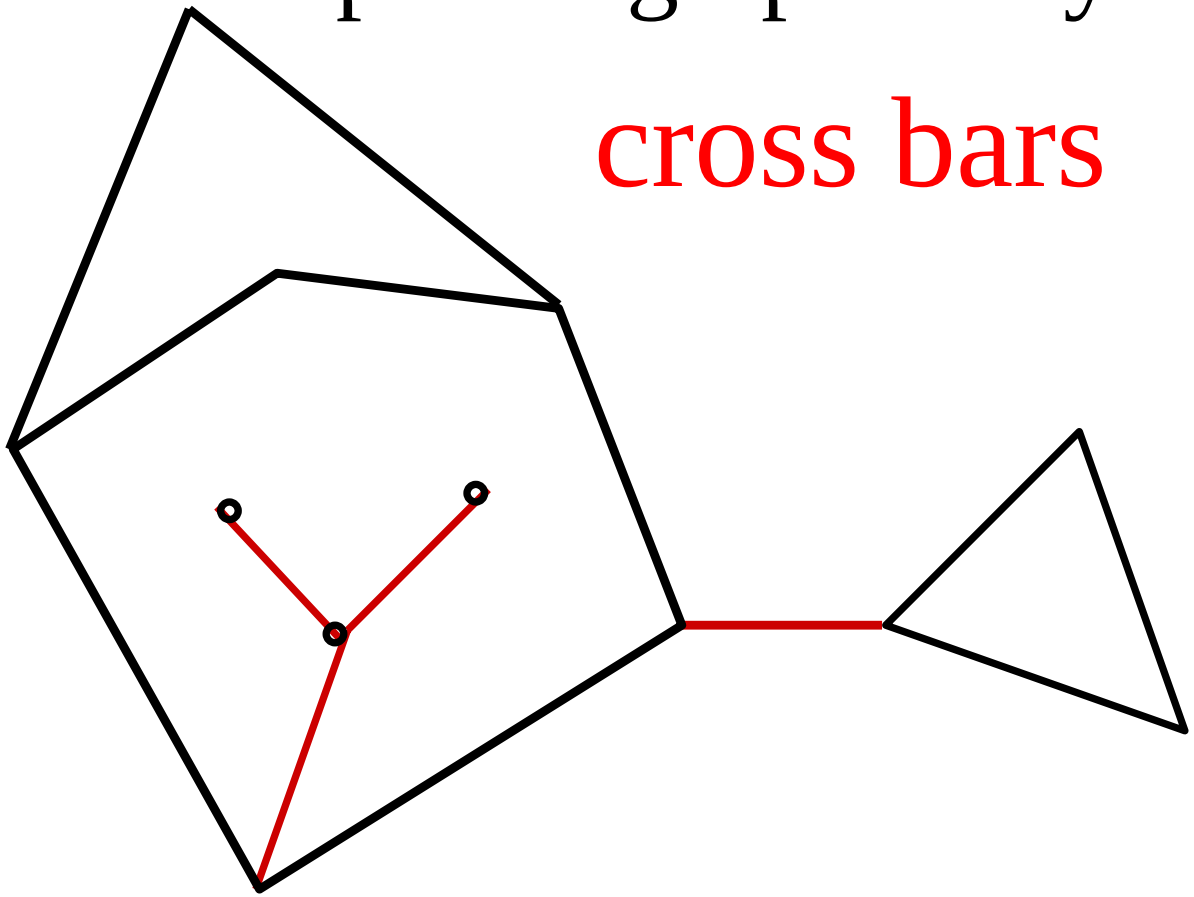
Planar Graphs

Maps are 2-connected planar graphs

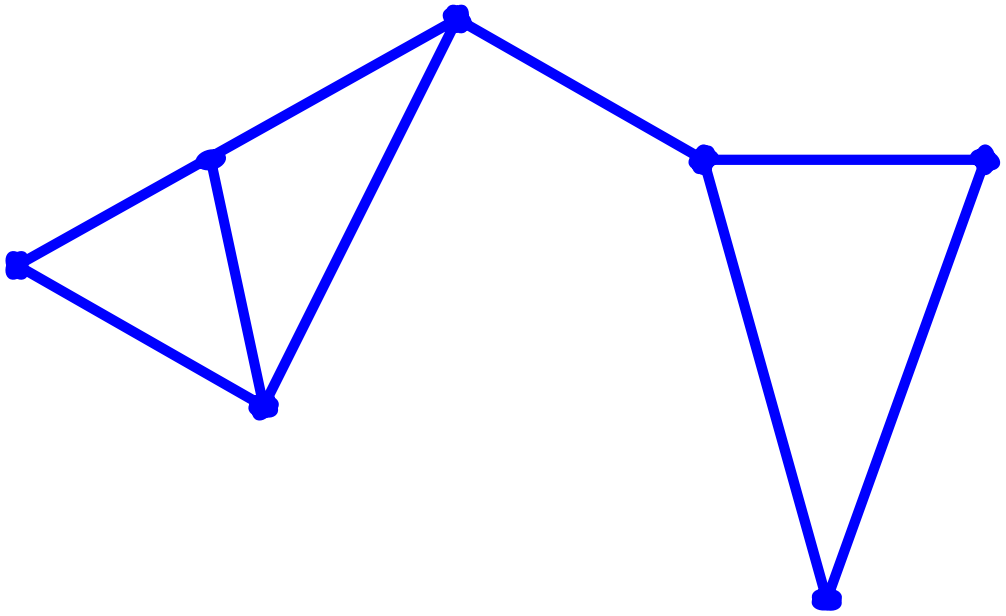
General connected planar graphs may have

dongles

cross bars

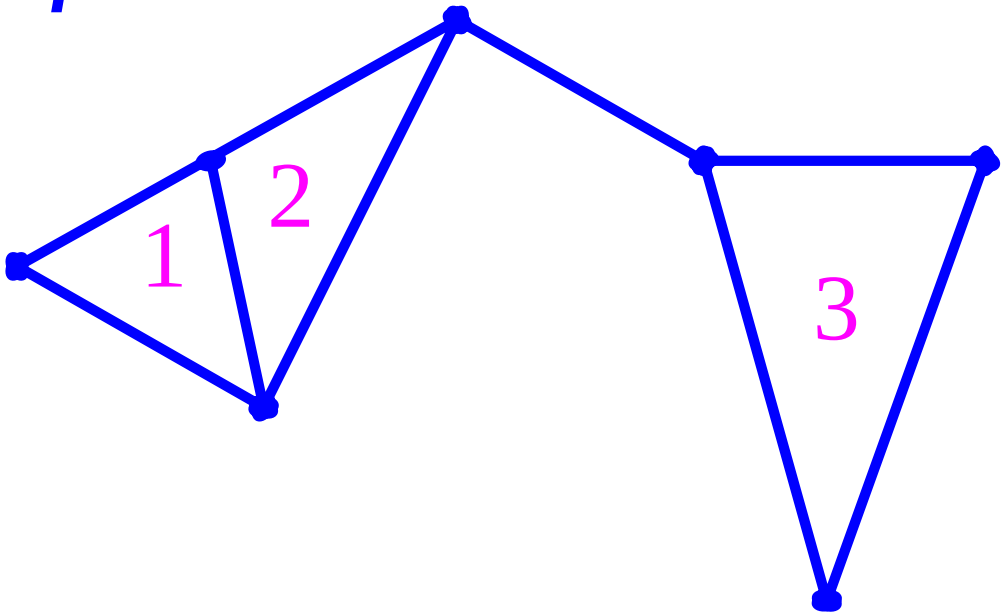


A Planar Graph



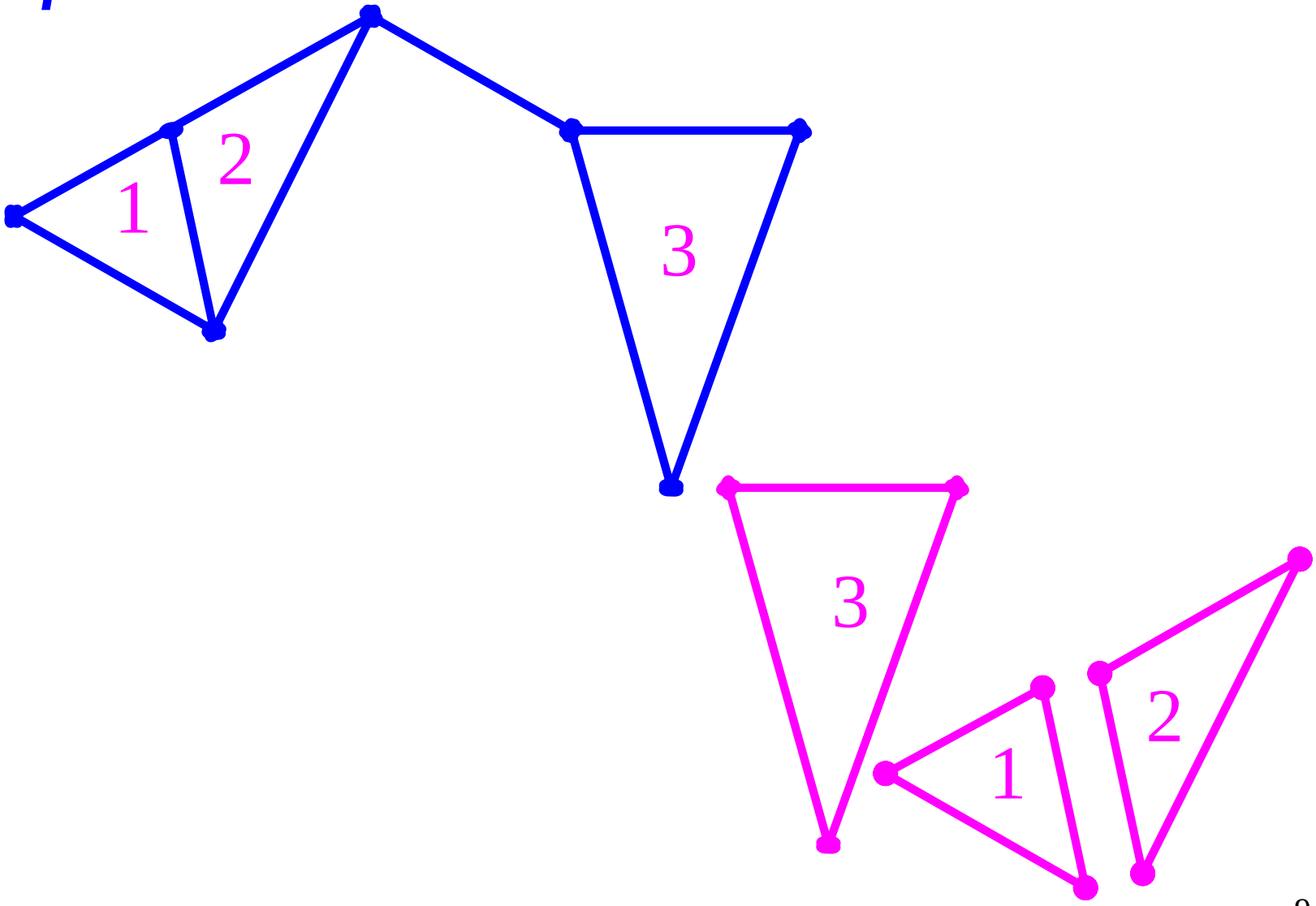
A Planar Graph

with 3 *faces*



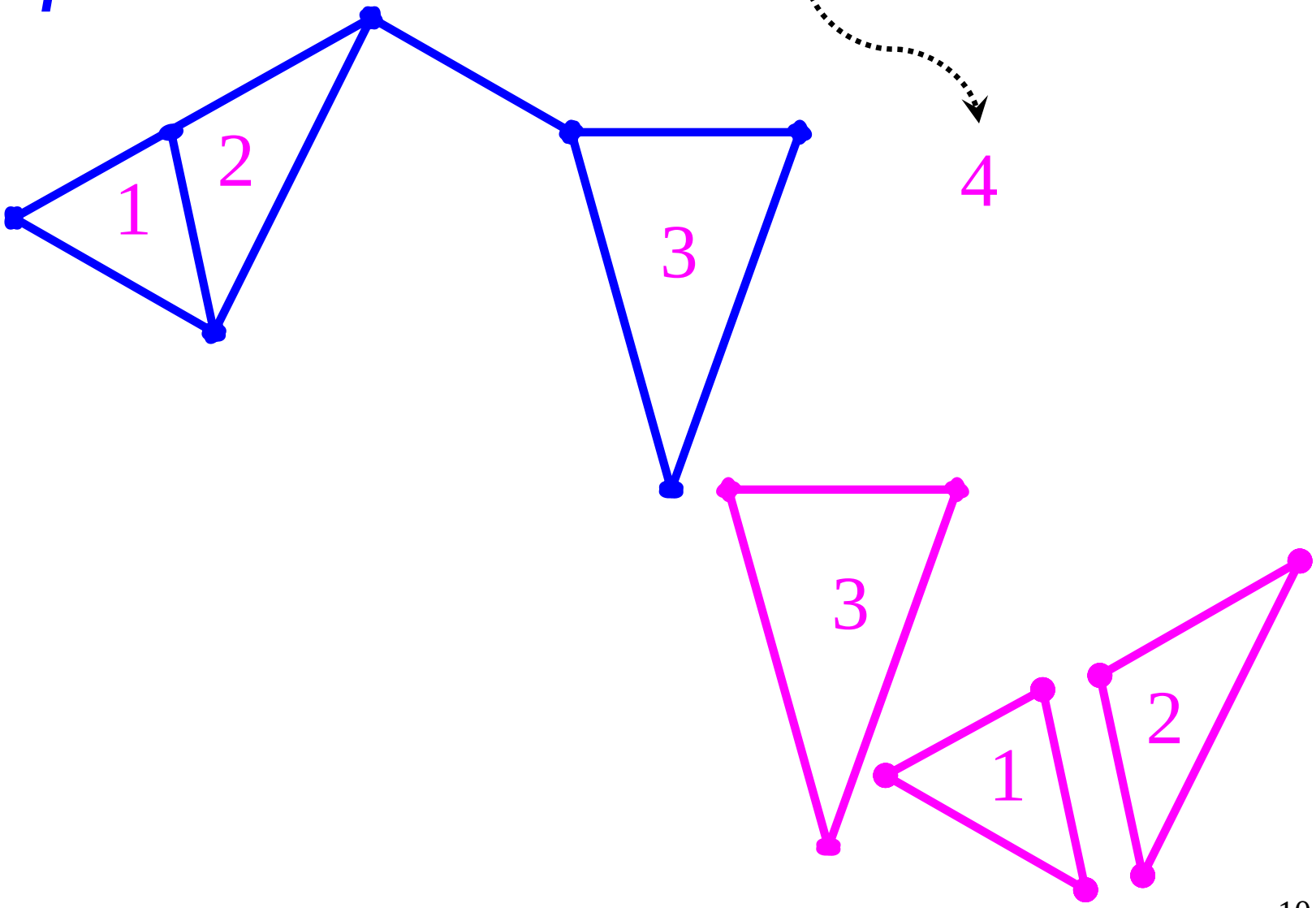
A Planar Graph

with 3 *faces*



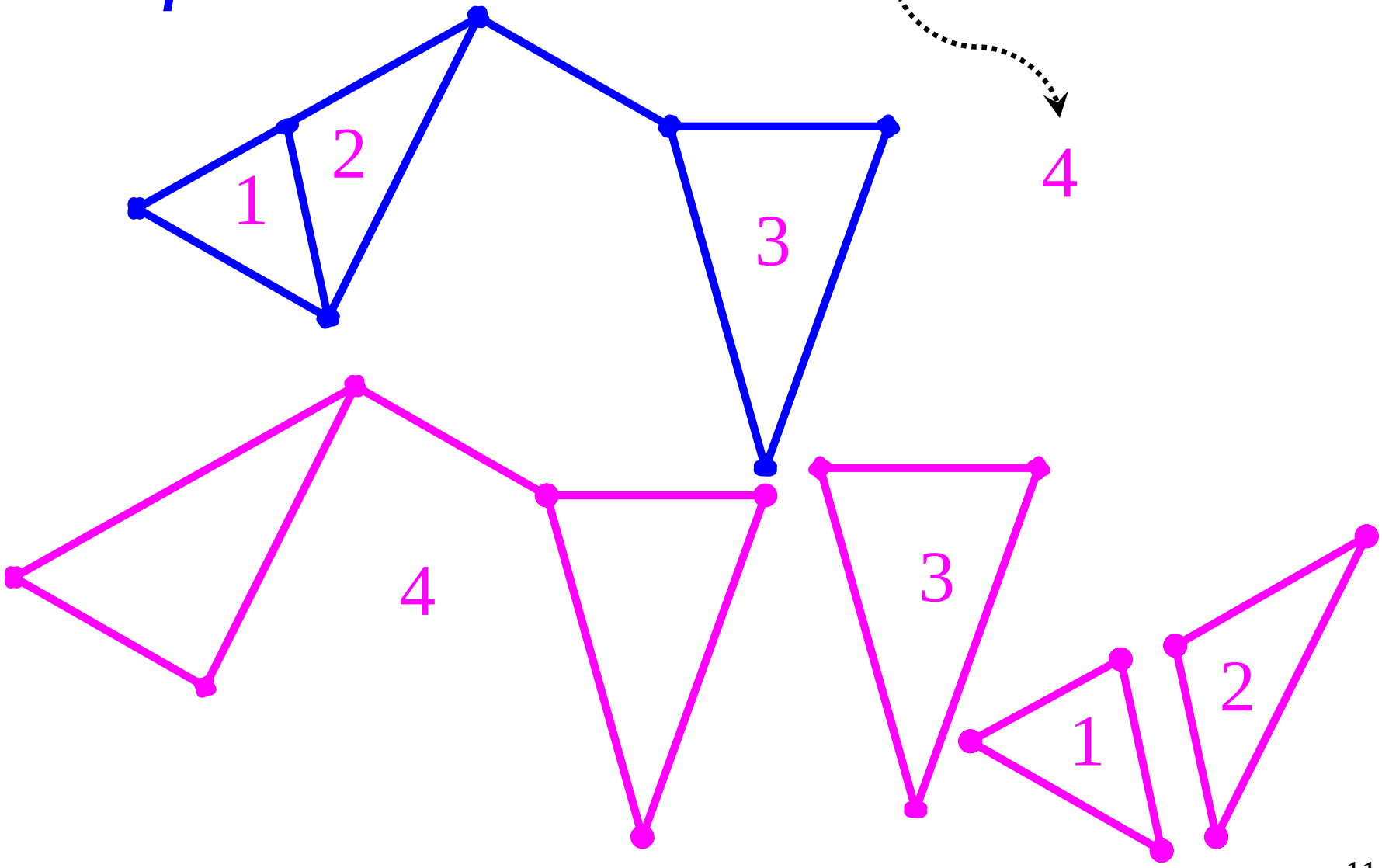
A Planar Graph

with 3 *faces* (wait! also the outer face)



A Planar Graph

with 4 *faces* (wait! also the outer face)



Drawing a Planar Graph

draw it edge by edge,
starting with a single vertex

Drawing a Planar Graph

draw it edge by edge,
starting with a single vertex
graph .

Drawing a Planar Graph

and record faces while drawing graph .

Drawing a Planar Graph

and record faces while drawing

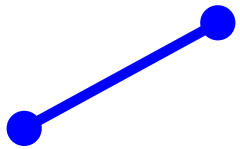
graph •

faces •

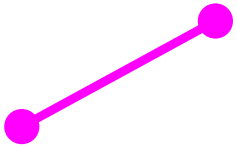
Planar Graphs

and record faces while drawing

graph



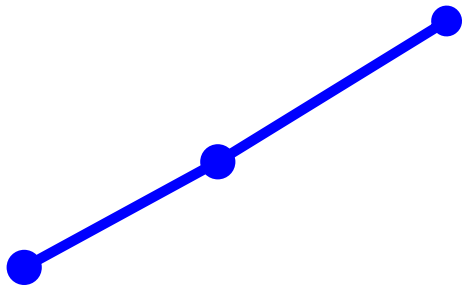
faces



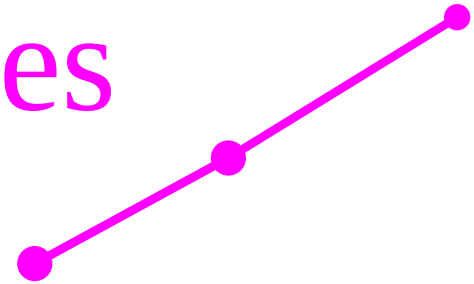
Planar Graphs

and record faces while drawing

graph



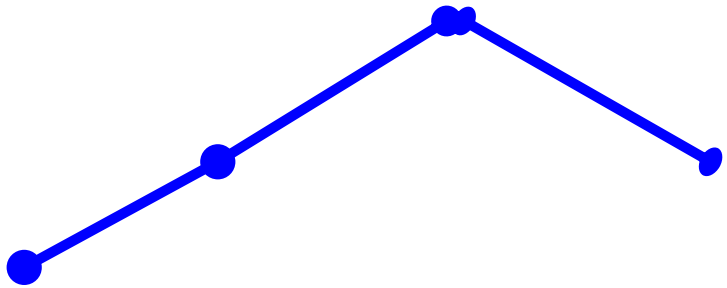
faces



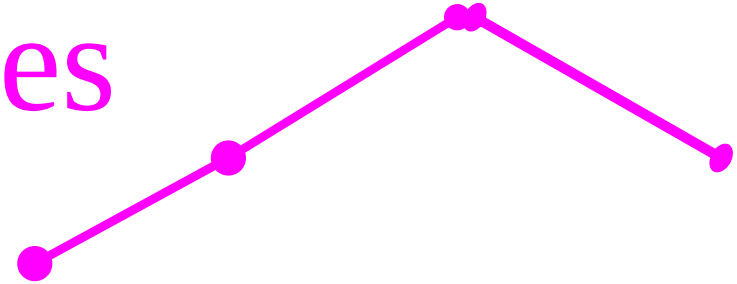
Planar Graphs

and record faces while drawing

graph



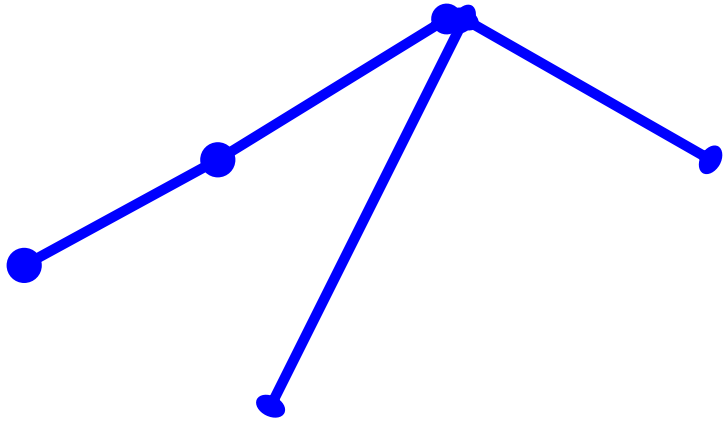
faces



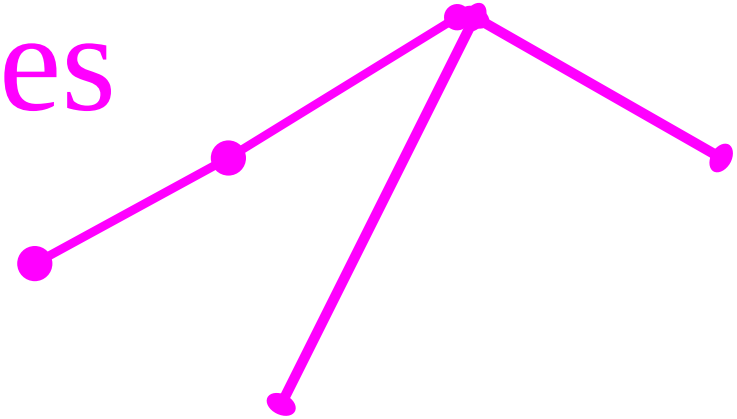
Planar Graphs

and record faces while drawing

graph



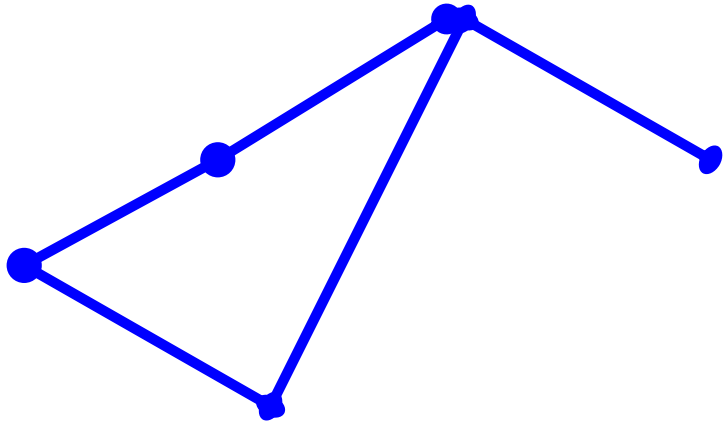
faces



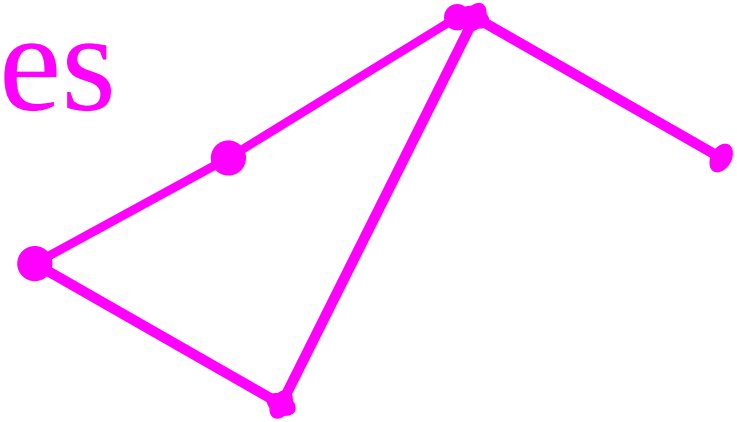
Planar Graphs

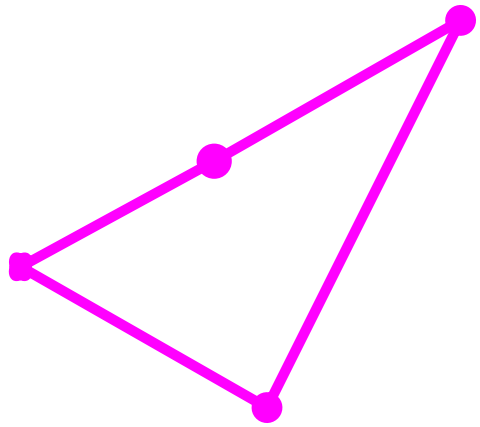
and record faces while drawing

graph



faces



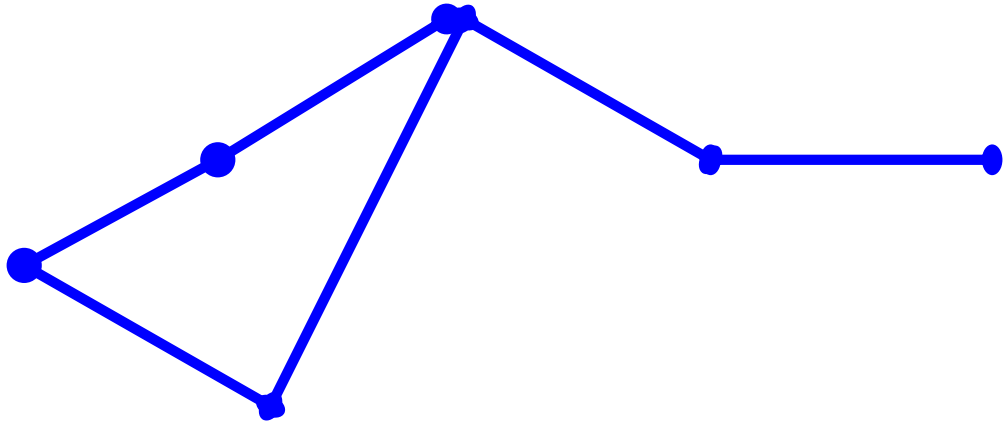


(the outer face)

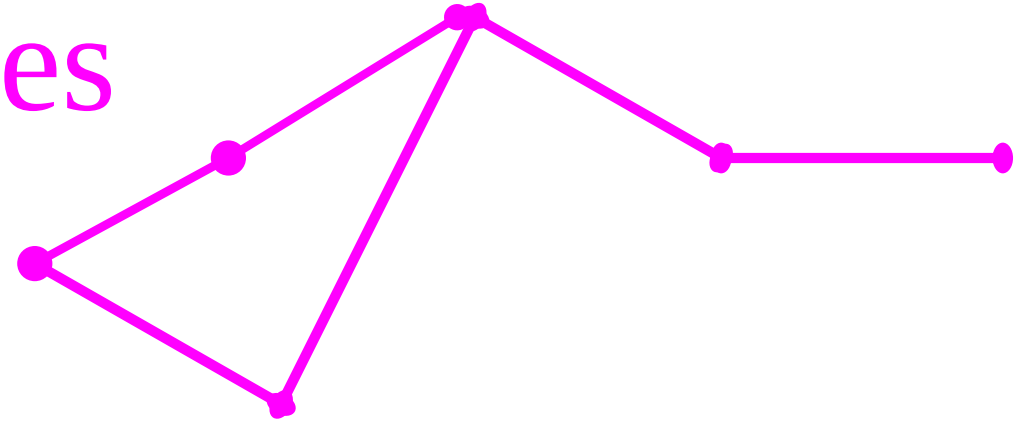
Planar Graphs

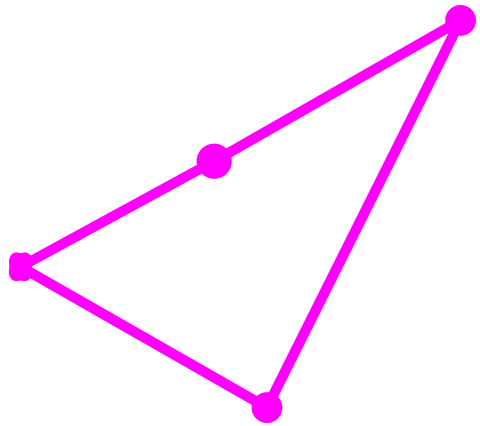
and record faces while drawing

graph



faces





(the outer face)

Planar Graphs

and record faces while drawing

graph

faces

(the outer face)

Planar Graphs

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faces

(the outer face)

Planar Graphs

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faces

(the outer face)

Planar Graphs

and record faces while drawing

graph

faces

(the outer face)

Recursive Definition of Faces

Precise rules defining the cycles that are the face boundaries of a Planar Drawing:

Recursive Definition of Faces

Start with a vertex •

Recursive Definition of Faces

Start with a vertex •

There is one face, whose boundary is the 0-length cycle consisting of this vertex.

Recursively adding an edge to a drawing

Two cases for connected graph:

1) Attach edge from vertex on a face to a new vertex.

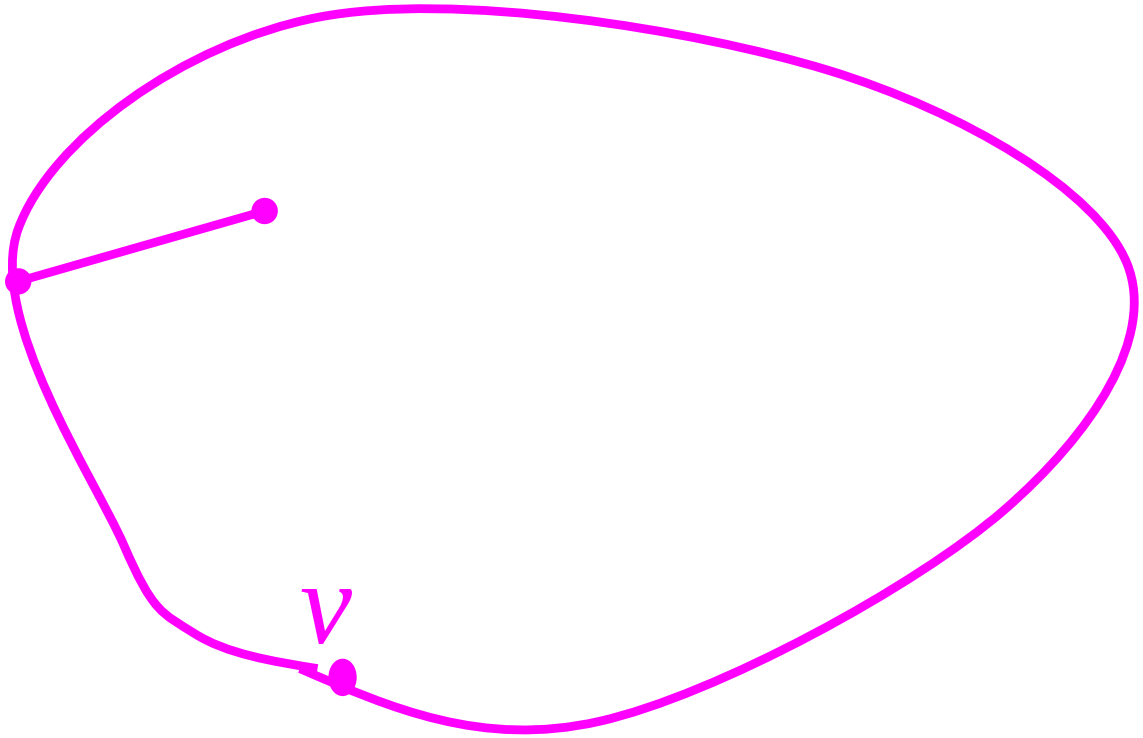
2) Attach edge between nonadjacent vertices on a face.

Face Creation Rule 1

1) choose vertex, v, on a face boundary

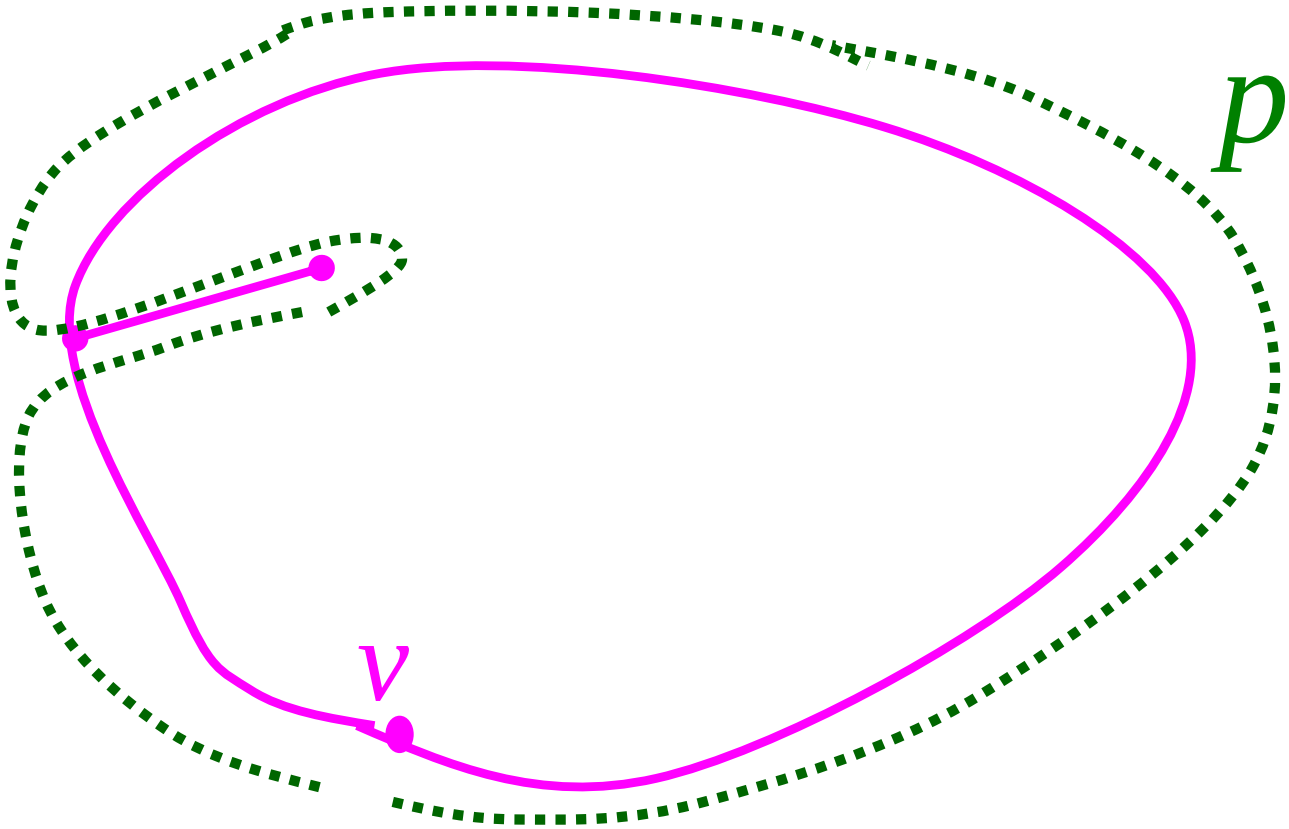
Face Creation Rule 1

1) choose vertex, v, on a face boundary



Face Creation Rule 1

1) choose vertex, *v*, on a face boundary



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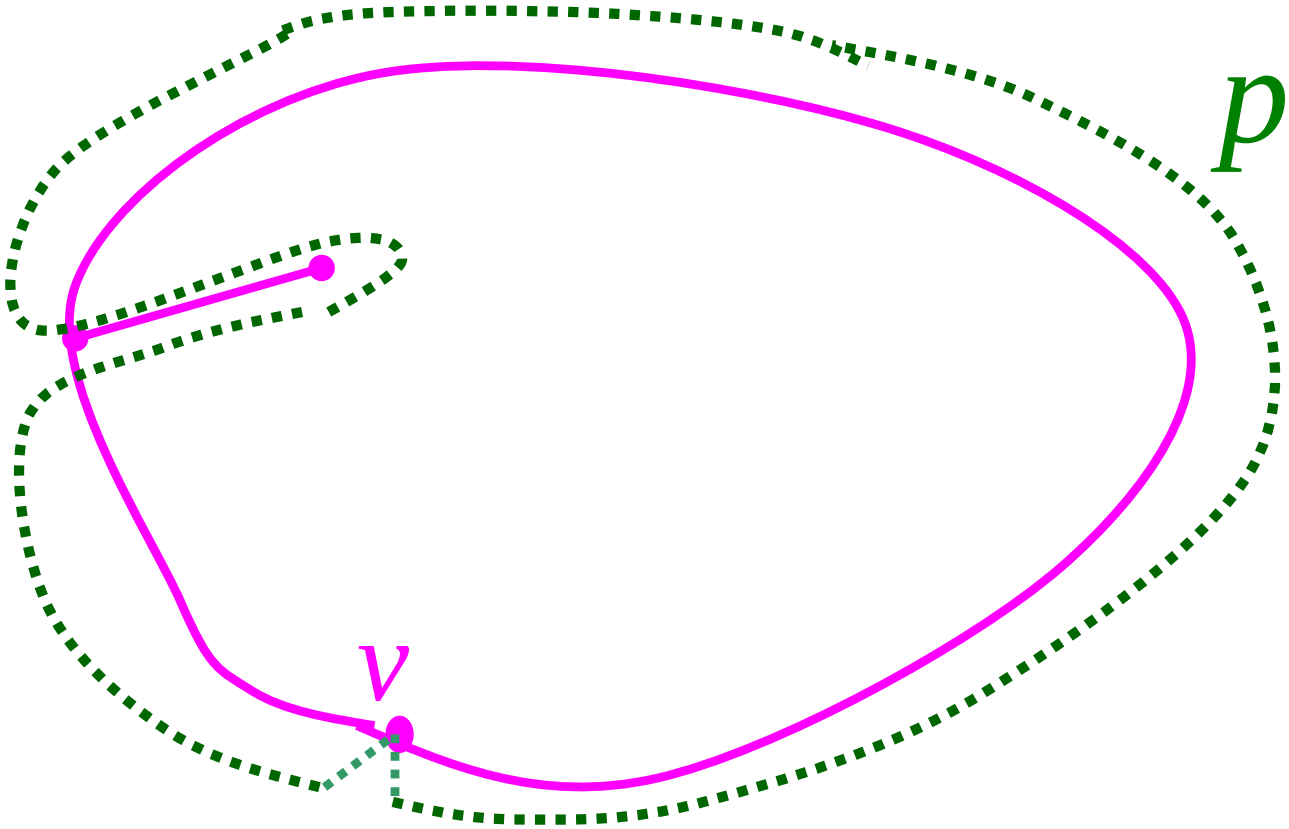

face boundary *vxv*

Face Creation Rule 1

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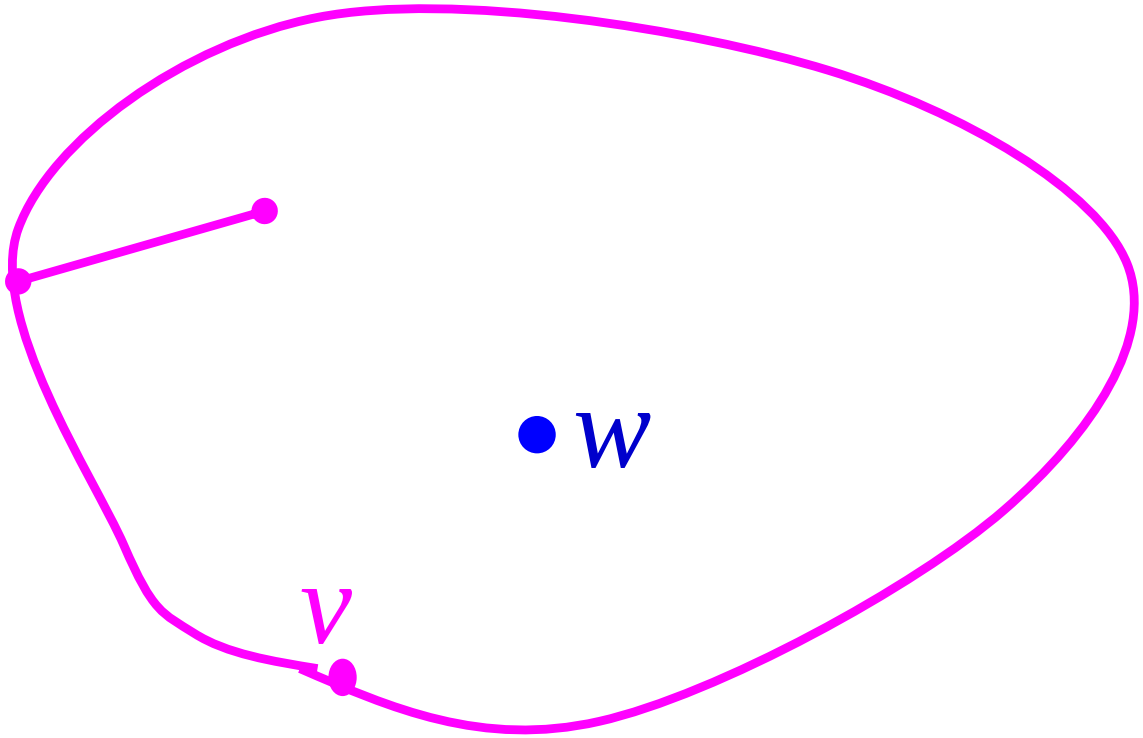
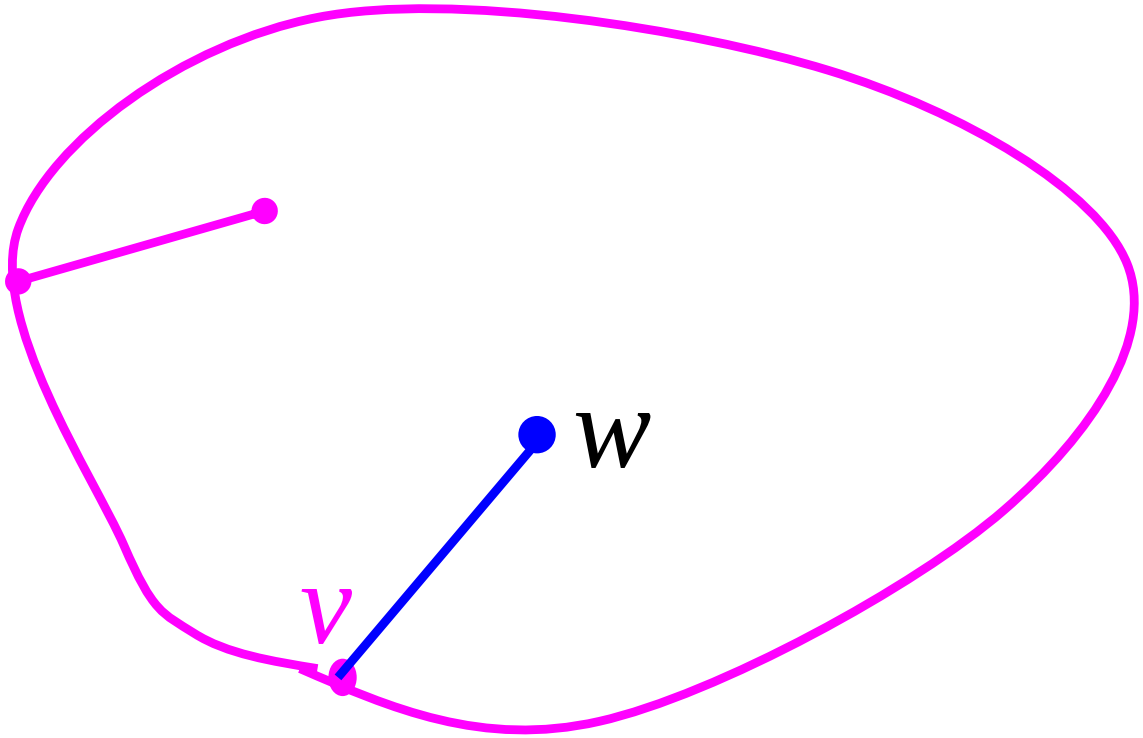


Create new vertex, w,

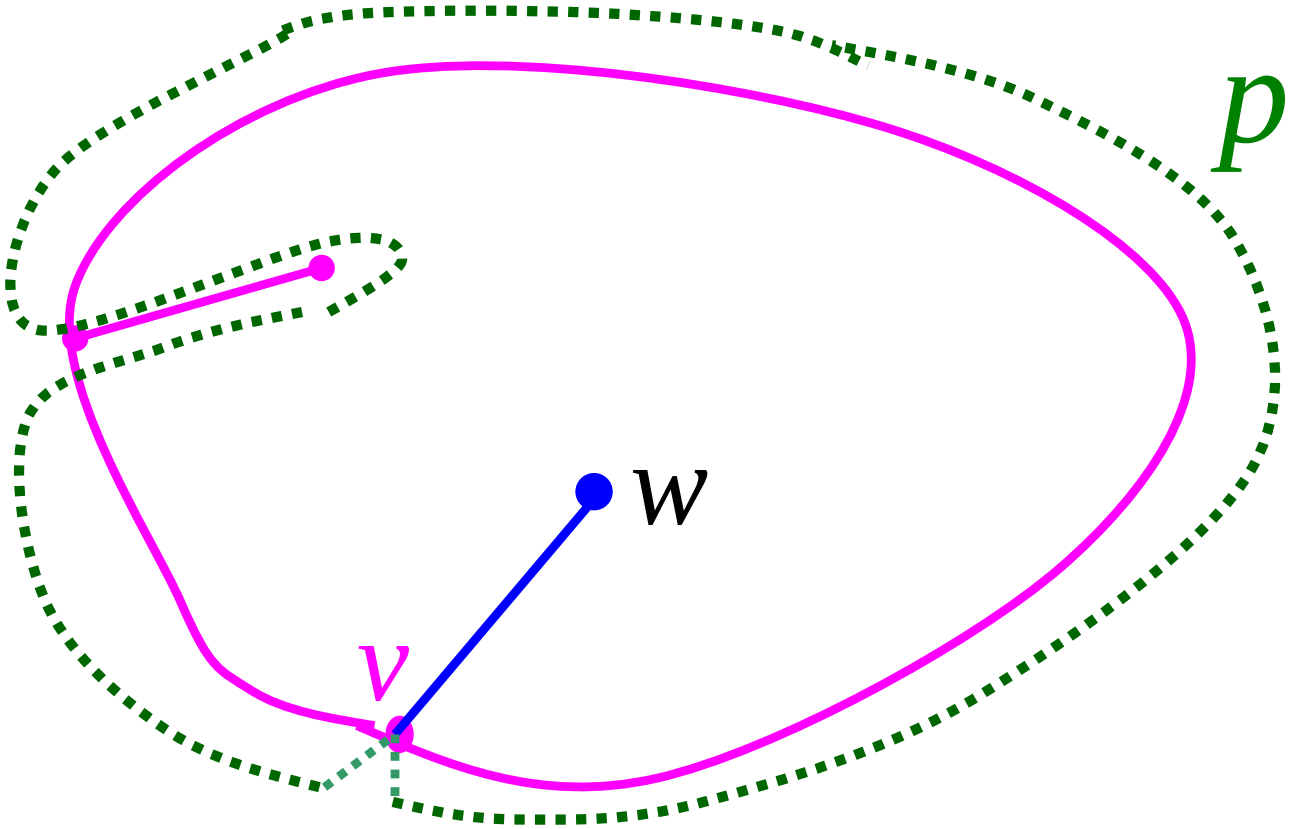
Face Creation Rule 1

1) choose vertex, v, on a face boundary



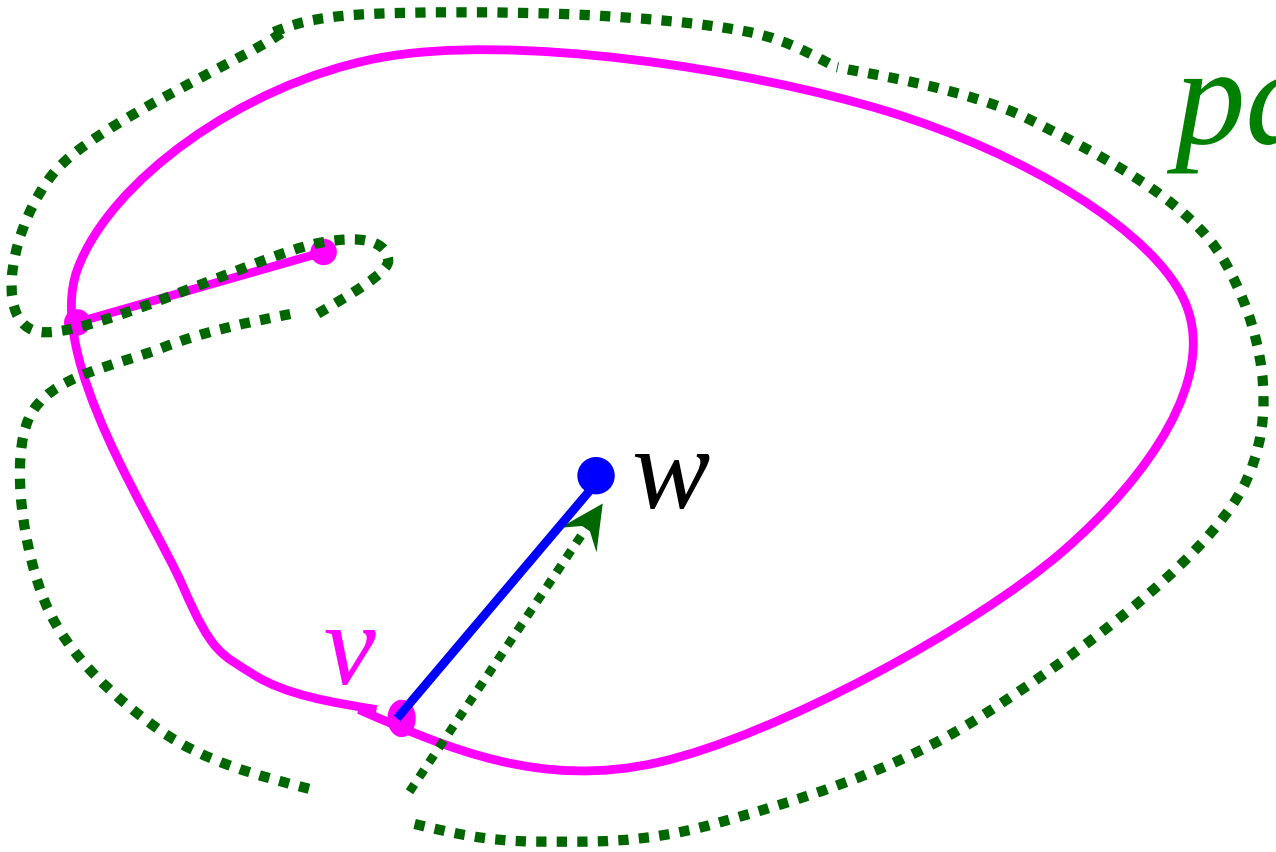
Create new vertex, w,
and add edge v--w

Face Creation Rule 1



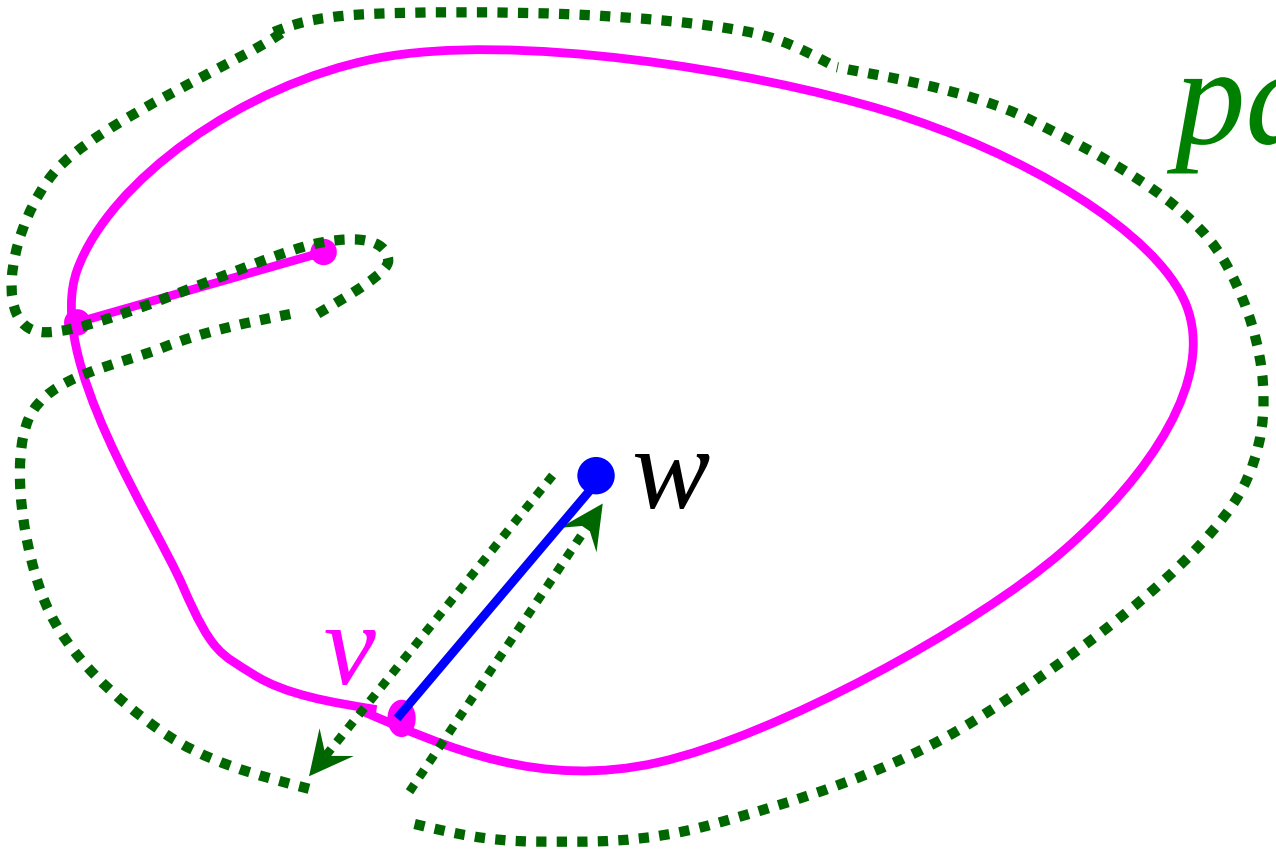
old face boundary ~~*vxv*~~

Face Creation Rule 1



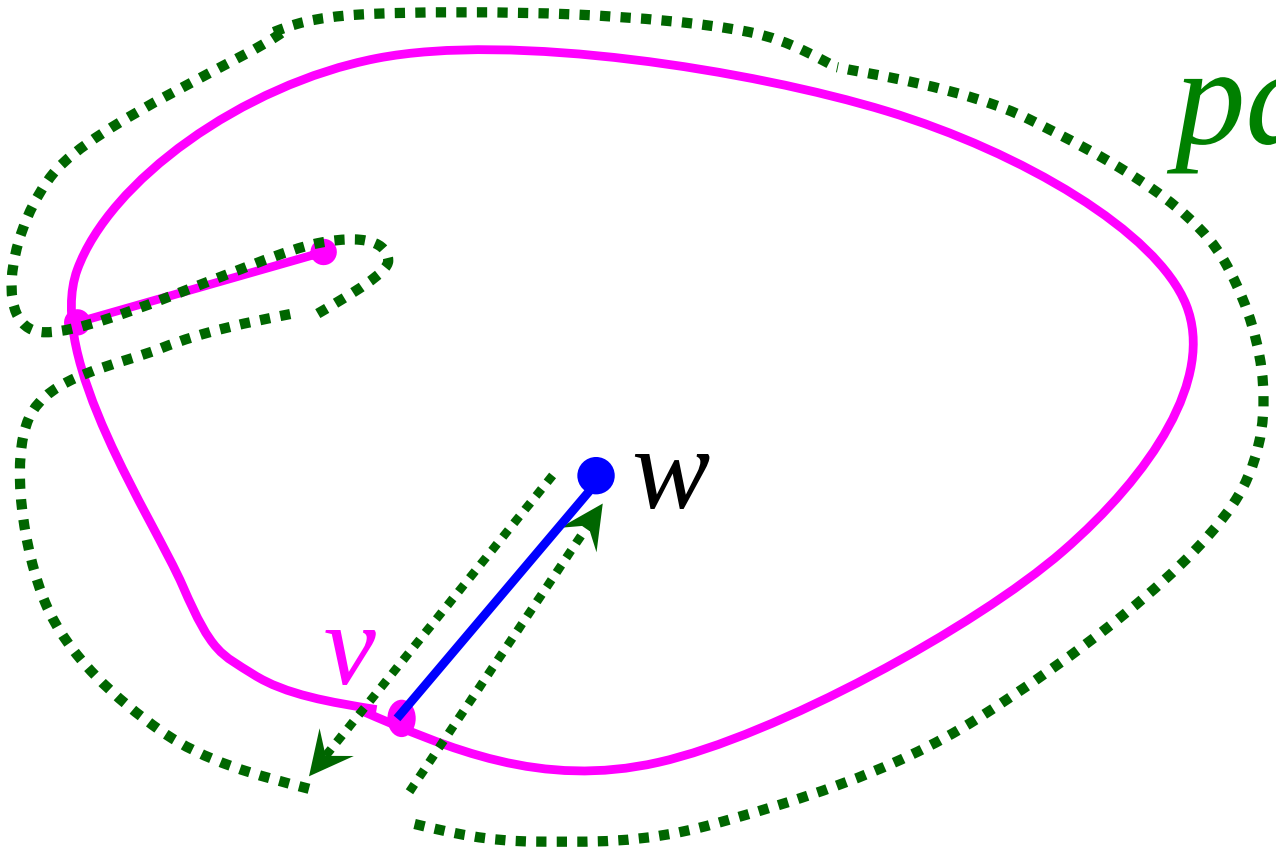
new face boundary

Face Creation Rule 1



new face boundary

Face Creation Rule 1



new face boundary *vwvxv*

Face Creation Rules

nothing else changes

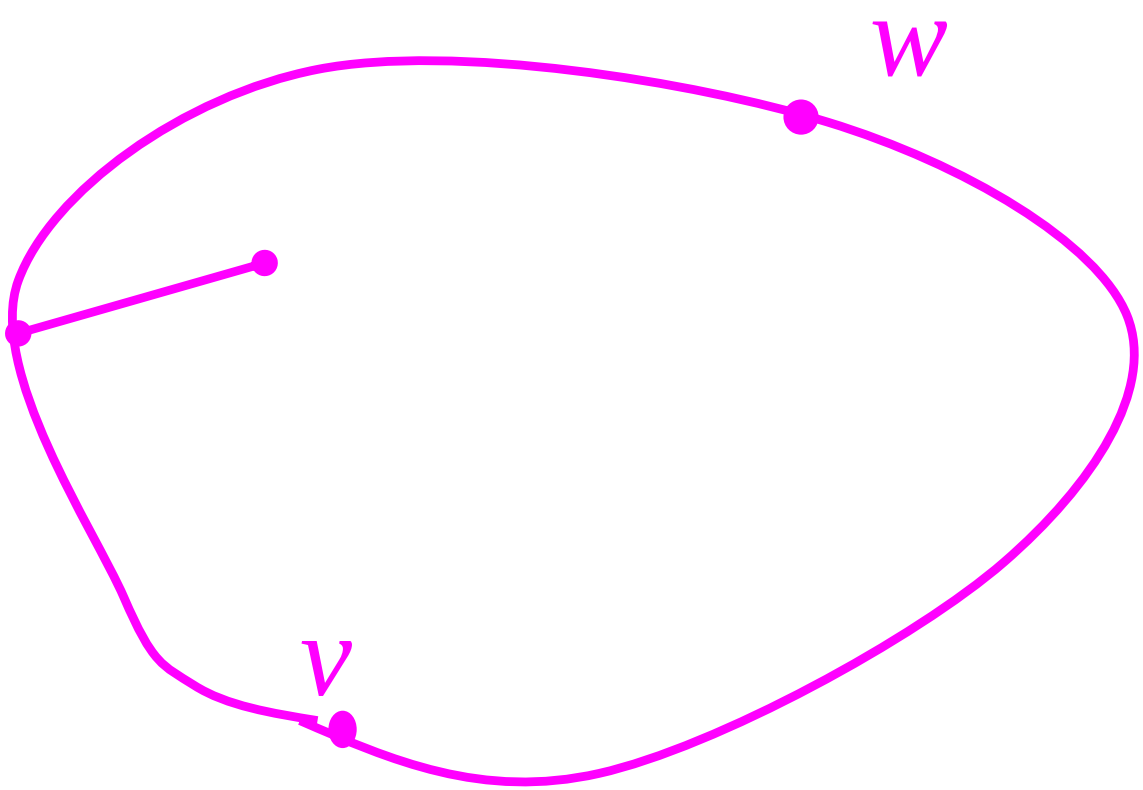
new face boundary *vwvxv*

Recursive Face Creation Rule 2

2) choose vertices v, w on a face boundary

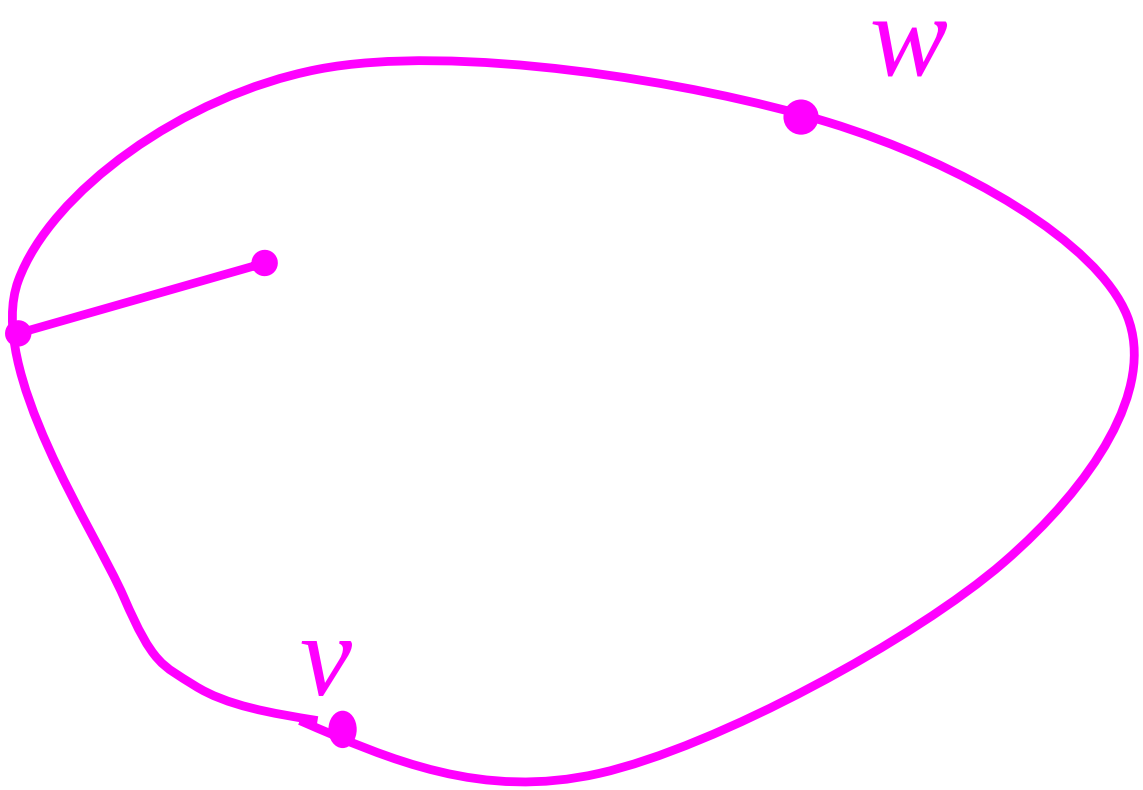
Face Creation Rule 2

2) choose vertices v, w on a face boundary



Face Creation Rule 2

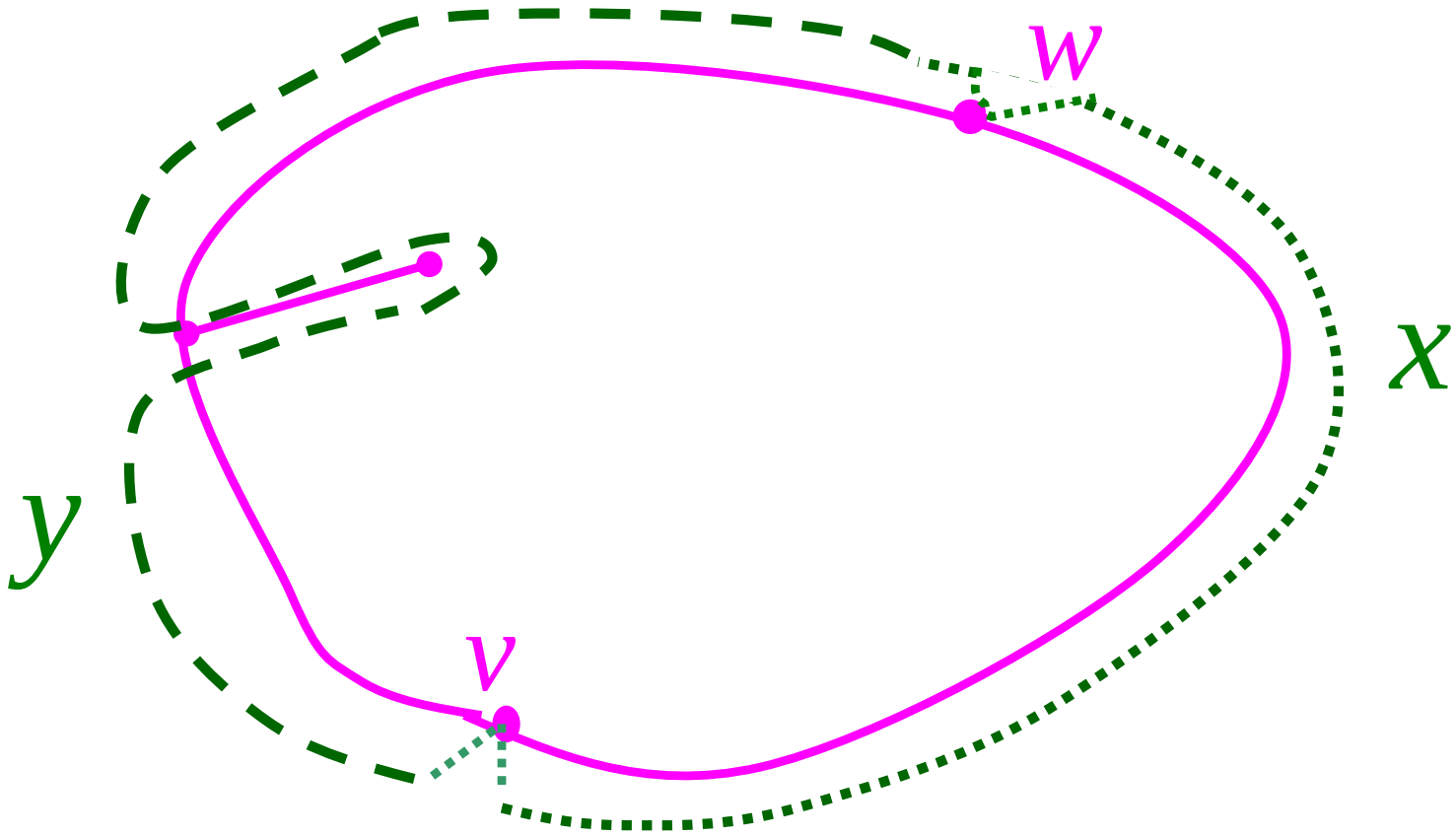
2) choose vertices v, w on a face boundary



with v, w, not adjacent

Face Creation Rule 2

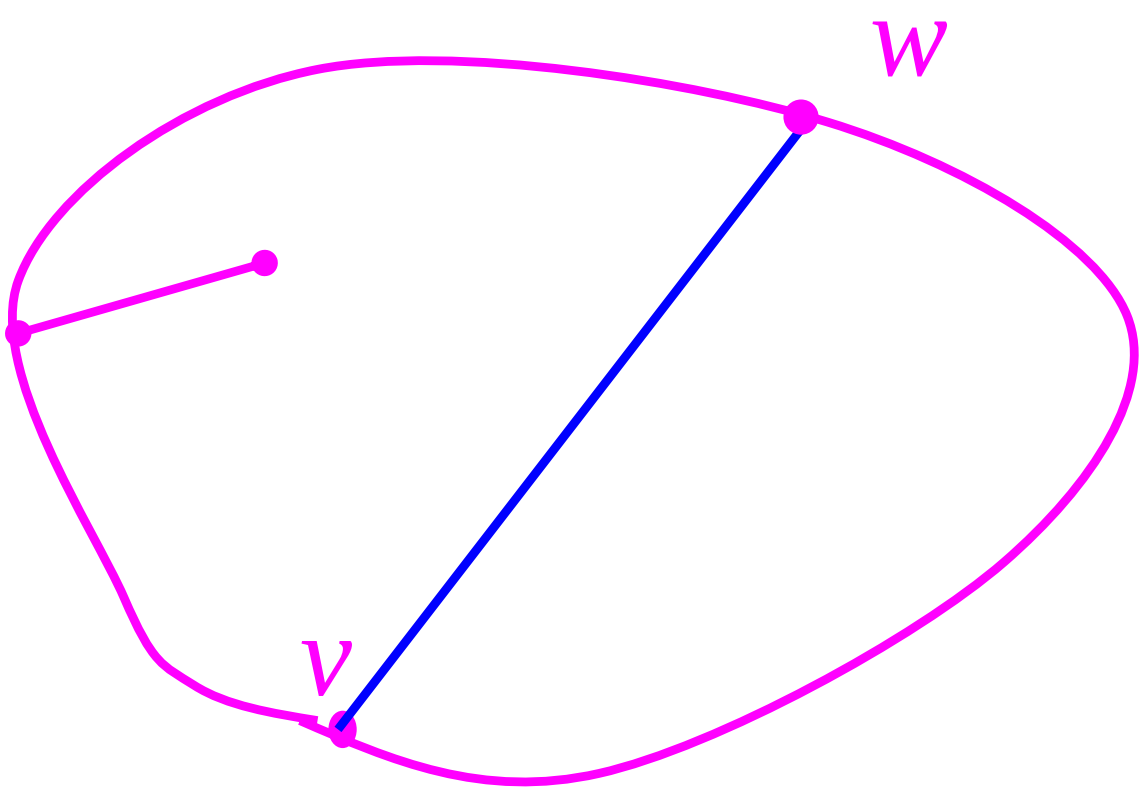
2) choose vertices v, w on a face boundary



face boundary $vywxv$

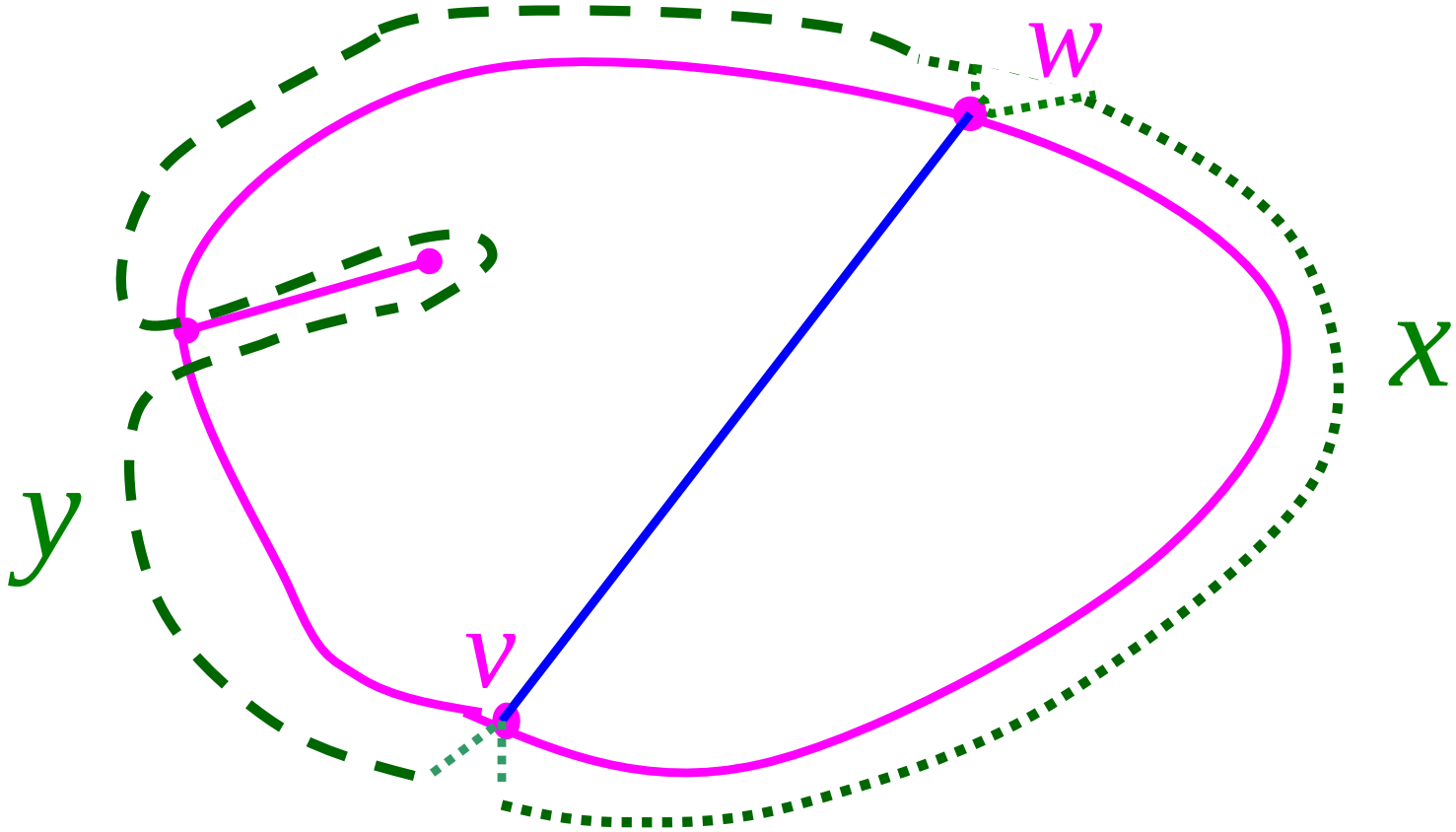
Face Creation Rule 2

2) choose vertices v, w on a face boundary



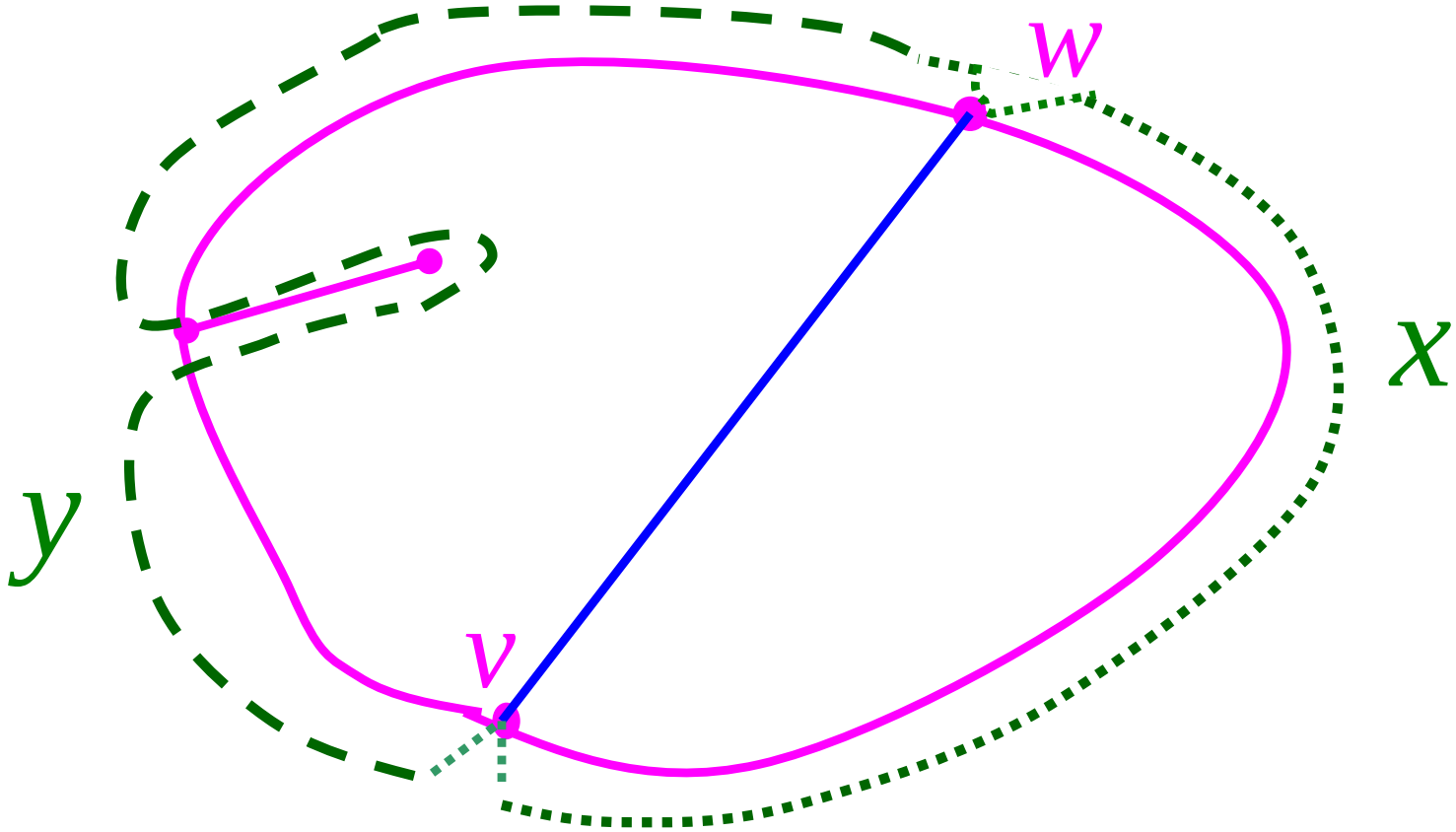
and add edge v--w

Face Creation Rule 2



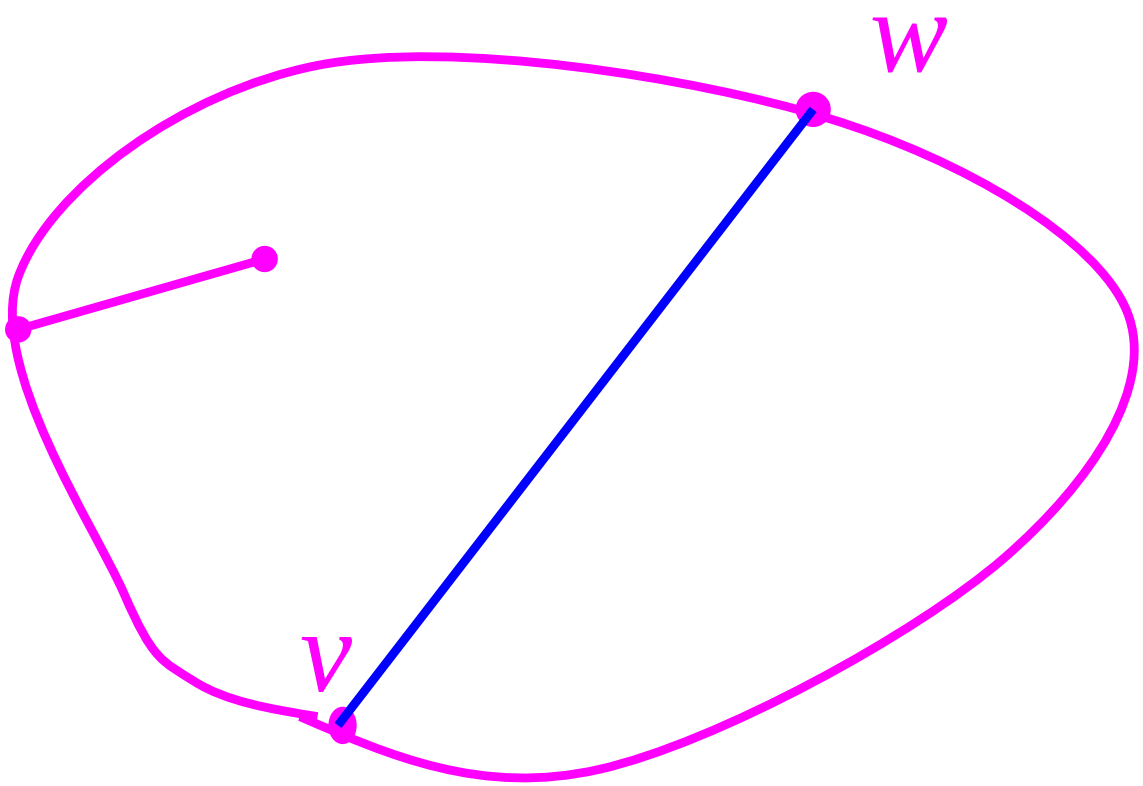
old face boundary *vywxv*

Face Creation Rule 2



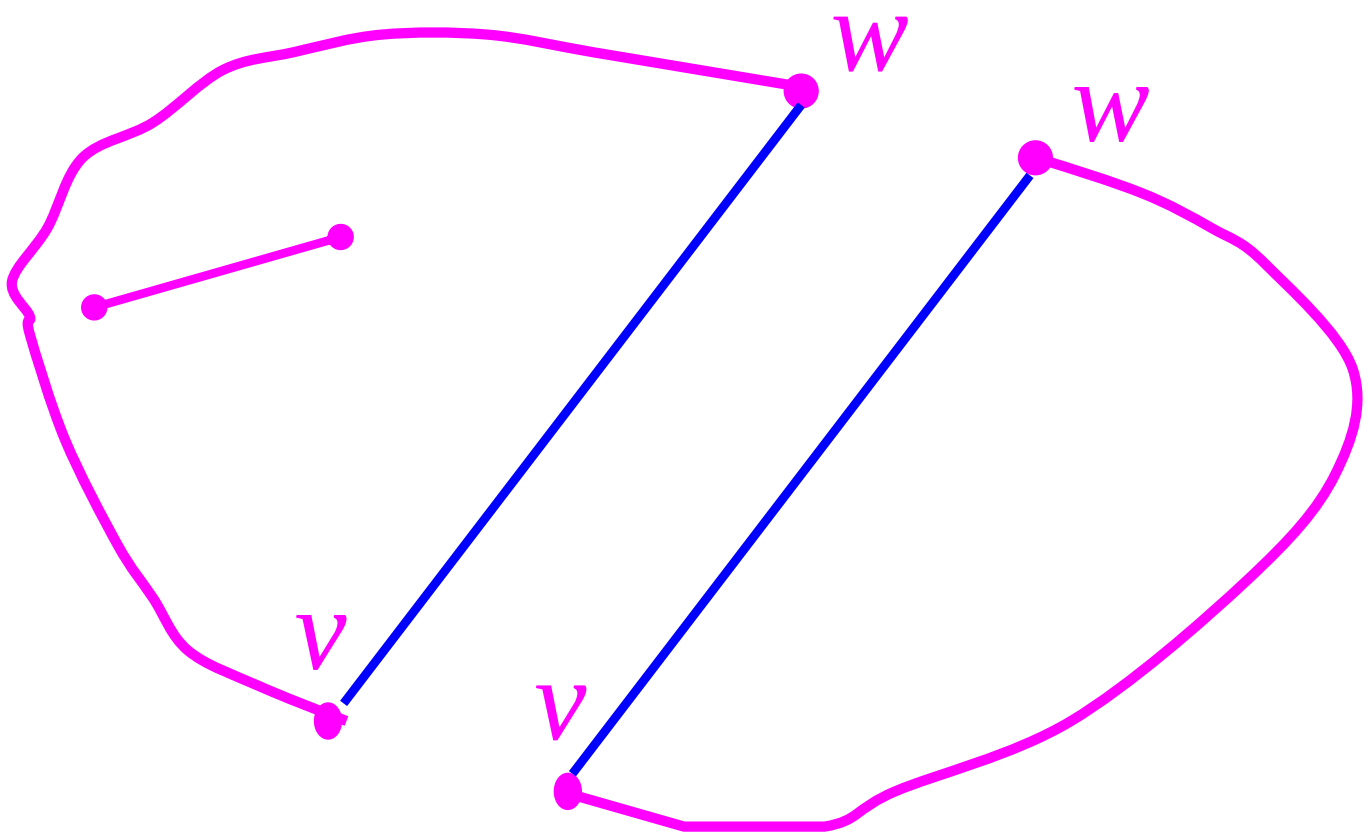
old face boundary *vywxv*

Face Creation Rule 2



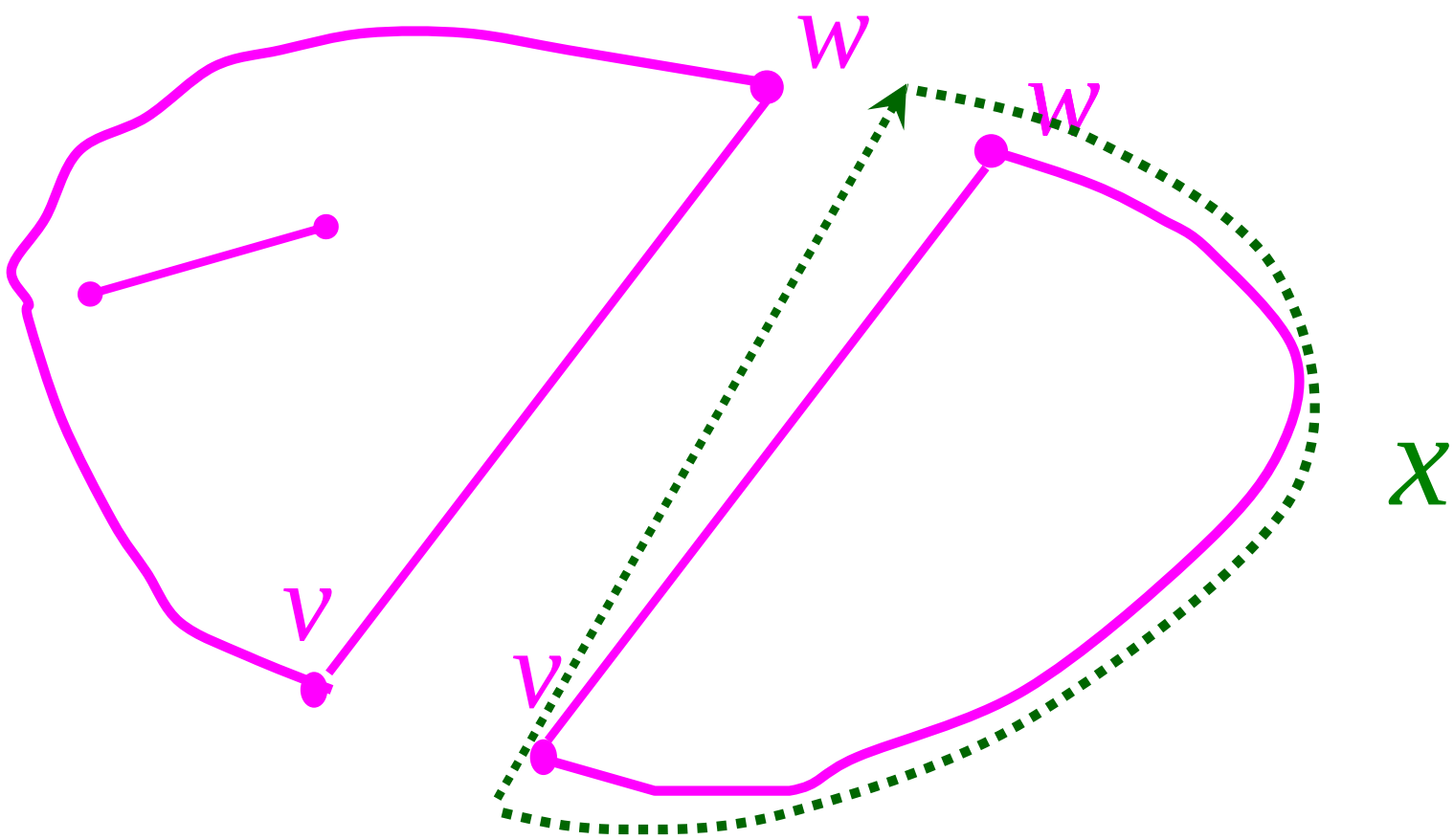
splits into 2 faces:

Face Creation Rule 2



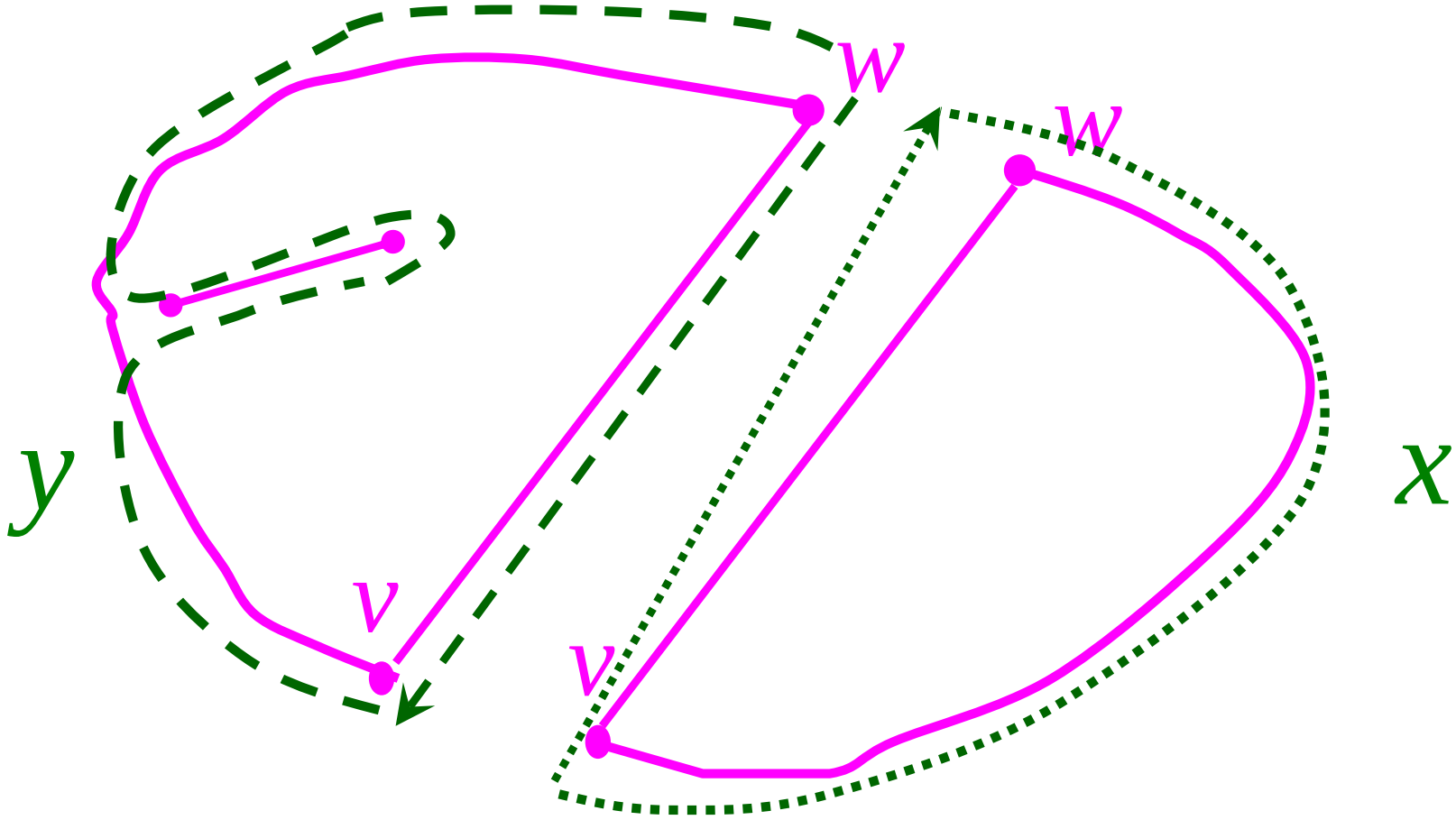
splits into 2 faces:

Face Creation Rule 2



splits into 2 faces: *vwxv*

Face Creation Rule 2



splits into 2 faces: *vwxv*, *vywv*

Face Creation Rules

nothing else changes

splits into 2 faces: *vwxv*, *vywv*

Recursive Definition of Faces

Every connected planar drawing is obtained by starting with a single vertex, and repeatedly applying Rules 1 & 2.

Induction on Drawings

Properties of planar drawings like Euler's formula can be proved by induction on the number of rule applications used to create a drawing.