In-Class Problems Week 4, Fri.

Problem 1. **(a)** For any vertex, v, in a graph, let $\widehat{v}$ be the set of vertices adjacent to v, that is,

$$\widehat{v} ::= \{v' \mid v\text{—}v' \text{ is an edge of the graph}\}.$$

Suppose f is an isomorphism from graph G to graph H. Carefully prove that $f(\widehat{v}) = \widehat{f(v)}$.

(b) Conclude that if G and H are isomorphic graphs, then for each $k \in \mathbb{N}$, they have the same number of degree k vertices.

Problem 2. For each of the following pairs of graphs, either define an isomomorphism between them, or prove that there is none. (We write ab as shorthand for a—b.)

(a)

$$G_1 \text{ with } V_1 = \{1,2,3,4,5,6\},\ E_1 = \{12,23,34,14,15,35,45\}$$
$$G_2 \text{ with } V_2 = \{1,2,3,4,5,6\},\ E_2 = \{12,23,34,45,51,24,25\}$$

(b)

$$G_1 \text{ with } V_1 = \{1,2,3,4,5,6\},\ E_1 = \{12,23,34,14,45,56,26\}$$
$$G_2 \text{ with } V_2 = \{a,b,c,d,e,f\},\ E_2 = \{ab,bc,cd,de,ae,ef,cf\}$$

(c)

$$G_1 \text{ with } V_1 = \{a,b,c,d,e,f,g,h\},\ E_1 = \{ab,bc,cd,ad,ef,fg,gh,he,dh,bf\}$$
$$G_2 \text{ with } V_2 = \{s,t,u,v,w,x,y,z\},\ E_2 = \{st,tu,uv,sv,wx,xy,yz,wz,sw,vz\}$$

Problem 3. Extra Problem.

(a) Exhibit three nonisomorphic, connected graphs with five vertices and four edges.

(b) Argue that every connected graph with five vertices and four edges is isomorphic to one of the three in part (a).