

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Electrical Engineering & Computer Science  
**6.041/6.431: Probabilistic Systems Analysis**  
(Quiz | Fall 2010)

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**Problem 1. (80 points)** In this problem:

- (i)  $X$  is a (continuous) uniform random variable on  $[0, 4]$ .
- (ii)  $Y$  is an exponential random variable, independent from  $X$ , with parameter  $\lambda = 2$ .

1. (10 points) Find the mean and variance of  $X - 3Y$ .
2. (10 points) Find the probability that  $Y \geq X$ .  
(Let  $c$  be the answer to this question.)
3. (10 points) Find the conditional joint PDF of  $X$  and  $Y$ , given that the event  $Y \geq X$  has occurred.  
(You may express your answer in terms of the constant  $c$  from the previous part.)
4. (10 points) Find the PDF of  $Z = X + Y$ .
5. (10 points) Provide a fully labeled sketch of the conditional PDF of  $Z$  given that  $Y = 3$ .
6. (10 points) Find  $\mathbf{E}[Z | Y = y]$  and  $\mathbf{E}[Z | Y]$ .
7. (10 points) Find the joint PDF  $f_{Z,Y}$  of  $Z$  and  $Y$ .
8. (10 points) A random variable  $W$  is defined as follows. We toss a fair coin (independent of  $Y$ ). If the result is “heads”, we let  $W = Y$ ; if it is tails, we let  $W = 2 + Y$ . Find the probability of “heads” given that  $W = 3$ .

**Problem 2. (30 points)** Let  $X, X_1, X_2, \dots$  be independent normal random variables with mean 0 and variance 9. Let  $N$  be a positive integer random variable with  $\mathbf{E}[N] = 2$  and  $\mathbf{E}[N^2] = 5$ . We assume that the random variables  $N, X, X_1, X_2, \dots$  are independent. Let  $S = \sum_{i=1}^N X_i$ .

1. (10 points) If  $\delta$  is a small positive number, we have  $\mathbf{P}(1 \leq |X| \leq 1 + \delta) \approx \alpha\delta$ , for some constant  $\alpha$ . Find the value of  $\alpha$ .
2. (10 points) Find the variance of  $S$ .
3. (5 points) Are  $N$  and  $S$  uncorrelated? Justify your answer.
4. (5 points) Are  $N$  and  $S$  independent? Justify your answer.

Each question is repeated in the following pages. Please write your answer on the appropriate page.

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2. **(10 points)** Find the probability that  $Y \geq X$ .  
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3. (10 points) Find the conditional joint PDF of  $X$  and  $Y$ , given that the event  $Y \geq X$  has occurred.  
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4. (10 points) Find the PDF of  $Z = X + Y$ .

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5. (10 points) Provide a fully labeled sketch of the conditional PDF of  $Z$  given that  $Y = 3$ .

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6. (10 points) Find  $\mathbf{E}[Z | Y = y]$  and  $\mathbf{E}[Z | Y]$ .

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7. (10 points) Find the joint PDF  $f_{Z,Y}$  of  $Z$  and  $Y$ .

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1. **(10 points)** If  $\delta$  is a small positive number, we have  $\mathbf{P}(1 \leq |X| \leq 1 + \delta) \approx \alpha\delta$ , for some constant  $\alpha$ . Find the value of  $\alpha$ .

2. **(10 points)** Find the variance of  $S$ .

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3. (5 points) Are  $N$  and  $S$  uncorrelated? Justify your answer.

4. (5 points) Are  $N$  and  $S$  independent? Justify your answer.

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6.041SC Probabilistic Systems Analysis and Applied Probability  
Fall 2013

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