LECTURE 1

- **Readings:** Sections 1.1, 1.2

Lecture outline

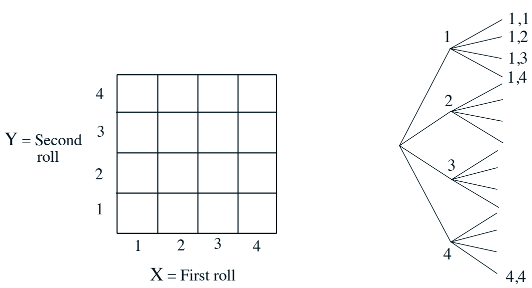
- Probability as a mathematical framework for:
  - reasoning about uncertainty
  - developing approaches to inference problems
- Probabilistic models
  - sample space
  - probability law
- Axioms of probability
- Simple examples

Sample space Ω

- "List" (set) of possible outcomes
- List must be:
  - Mutually exclusive
  - Collectively exhaustive
- Art: to be at the "right" granularity

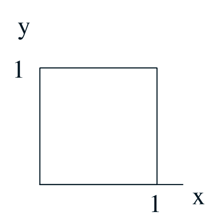
Sample space: Discrete example

- Two rolls of a tetrahedral die
  - Sample space vs. sequential description



Sample space: Continuous example

$\Omega = \{(x, y) \mid 0 \leq x, y \leq 1\}$



Probability axioms

- **Event:** a subset of the sample space
- Probability is assigned to events

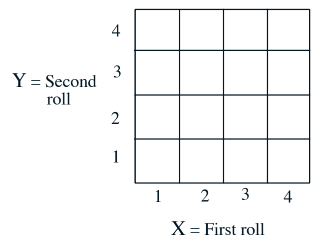
Axioms:

1. **Nonnegativity:** $\mathbf{P}(A) \geq 0$
2. **Normalization:** $\mathbf{P}(\Omega) = 1$
3. **Additivity:** If $A \cap B = \emptyset$, then $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B)$

- $$\begin{aligned}\mathbf{P}(\{s_1, s_2, \ldots, s_k\}) &= \mathbf{P}(\{s_1\}) + \cdots + \mathbf{P}(\{s_k\}) \\ &= \mathbf{P}(s_1) + \cdots + \mathbf{P}(s_k)\end{aligned}$$

- Axiom 3 needs strengthening
- Do weird sets have probabilities?

Probability law: Example with finite sample space



- Let every possible outcome have probability 1/16
  - $\mathbf{P}((X, Y)$ is (1,1) or (1,2)$) =$
  - $\mathbf{P}(\{X = 1\}) =$
  - $\mathbf{P}(X + Y$ is odd$) =$
  - $\mathbf{P}(\min(X, Y) = 2) =$

Discrete uniform law

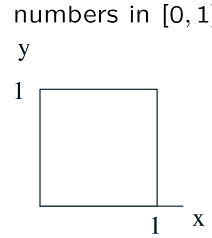
- Let all outcomes be equally likely
- Then,

$$\mathbf{P}(A) = \frac{\text{number of elements of } A}{\text{total number of sample points}}$$

- Computing probabilities $\equiv$ counting
- Defines fair coins, fair dice, well-shuffled card decks

Continuous uniform law

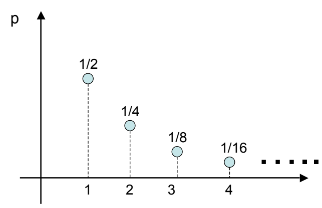
- Two "random" numbers in $[0, 1]$.



- **Uniform** law: Probability = Area
  - $\mathbf{P}(X + Y \leq 1/2) = ?$
  - $\mathbf{P}(\,(X, Y) = (0.5, 0.3)\,)$

Probability law: Ex. w/countably infinite sample space

- Sample space: $\{1, 2, \ldots\}$
  - We are given $\mathbf{P}(n) = 2^{-n}$, $n = 1, 2, \ldots$
  - Find $\mathbf{P}$(outcome is even)



$$\mathbf{P}(\{2, 4, 6, \ldots\}) = \mathbf{P}(2) + \mathbf{P}(4) + \cdots = \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdot$$

- Countable additivity axiom (needed for this calculation):
  If $A_1, A_2, \ldots$ are disjoint events, then:

$$\mathbf{P}(A_1 \cup A_2 \cup \cdots) = \mathbf{P}(A_1) + \mathbf{P}(A_2) + \cdots$$