6.041SC Probabilistic Systems Analysis and Applied Probability, Fall 2013 Transcript – Recitation: A Random Walker

In this problem, we'll be working with a object called random walk, where we have a person on the line-- or a tight rope, according to the problem. Let's start from the origin, and each time step, it would randomly either go forward or backward with certain probability. In our case, with probability P, the person would go forward, and 1 minus P going backwards.

Now, the walk is random in the following sense-- that the choice going forward or backward in each step is random, and it's completely independent from all past history. So let's look at the problem. It has three parts. In the first part, we'd like to know what's the probability that after two steps the person returns to the starting point, which in this case is 0?

Now, throughout this problem, I'm going to be using the following notation. F indicates the action of going forward and B indicates the action of going backwards. A sequence says F and B implies the sample that the person first goes forward, and then backwards.

If I add another F, it will mean, forward, backward, forward again. OK? So in order for the person to go somewhere after two steps and return to the origin, the following must happen. Either the person went forward followed by backward, or backward followed by forward.

And indeed, this event-- namely, the union of these two possibilities-- defines the event of interest in our case. And we'd like to know what's the probability of A, which we'll break down into the probability of forward, backward, backward, forward.

Now, forward, backward and backward, forward-- they are two completely different outcomes. And we know that because they're disjoint, this would just be the sum of the two probabilities-- plus probability of backward/forward. Here's where the independence will come in.

When we try to compute the probability of going forward and backward, because the action-- each step is completely independent from the past, we know this is the same as saying, in the first step, we have probability P of going forward, in the next step, probability 1 minus P of going backwards.

We can do so-- namely, writing the probability of forward, backward as a product of going forward times the probability of going backwards, because these actions are independent. And similarly, for the second one, we have going backwards first, times going forward the second time.

Adding these two up, we have 2 times P times 1 minus P. And that will be the answer to the first part of the problem. In the second part of the problem, we're interested in the probability that after three steps, the person ends up in position 1, or one step forward compared to where he started.

Now, the only possibilities here are that among the three steps, exactly two steps are forward, and one step is backwards, because otherwise there's no way the person will end up in position 1. To do so, there, again, are three possibilities in which we go forward, forward, backward, or forward, backward, forward, or backward, forward, forward.

And that exhausts all the possibilities that the person can end up in position 1 after three steps. And we'll define the collection of all these outcomes as event C. The probability of event C-- same as before-- is simply the sum of the probability of each individual outcome.

Now, based on the independence assumption that we used before, each outcome here has the same probability, which is equal to P squared times 1 minus P. The P squared comes from the fact that two forward steps are taken, and 1 minus P, the probability of that one backwards step.

And since there are three of them, we multiply 3 in front, and that will give us the probability. In the last part of the problem, we're asked to compute that, conditional on event C already took place, what is the probability that the first step he took was a forward step?

Without going into the details, let's take a look at the C, in which we have three elements, and only the first two elements correspond to a forward step in the first step. So we can define event D as simply the first two outcomes-- forward, forward, backward, and forward, backward, forward.

Now, the probability we're interested in is simply probability of D conditional on C. We'd write it out using the law of conditional probability-- D intersection C conditional on C. Now, because D is a subset of C, we have probability of D divided by the probability of C.

Again, because all samples here have the same probability, all we need to do is to count the number of samples here, which is 2, and divide by the number of samples here, which is 3. So we end up with 2 over 3. And that concludes the problem. See you next time.

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