

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
 Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
 (Spring 2006)

Tutorial 11
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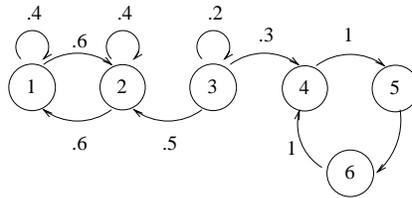
1. (Problem 5.14) Each morning as you pull out of your driveway you would like to make a U-turn rather than drive around the block. Unfortunately, U-turns are illegal in your neighborhood, and police cars drive by according to a Poisson process of rate λ . You decide to make a U-turn when you can see that the road will be clear of police cars for a time τ . Let N be the number of police cars you see before you make the U-turn.

a) Find $\mathbf{E}[N]$

b) Find the conditional expectation of the time elapsed between police cars $n - 1$ and n , given that $N \geq n$.

c) Find the expected time that you wait until you make a U-turn. *Hint:* Condition on N .

2. The Markov chain shown below is in state 3 immediately before the first trial.



- (a) Indicate which states, if any, are recurrent, transient, and periodic.
- (b) Find the probability that the process is in state 3 after n trials.
- (c) Find the expected number of trials up to and including the trial on which the process leaves state 3.
- (d) Find the probability that the process never enters state 1.
- (e) Find the probability that the process is in state 4 after 10 trials.
- (f) Given that the process is in state 4 after 10 trials, find the probability that the process was in state 4 after the first trial.