

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Tutorial 10
April 27-28, 2006

1. (Problem 5.3) A computer system carries out tasks submitted by two users. time is divided into slots. A slot can be idle, with probability $p_I = 1/6$, and busy with probability $p_B = 5/6$. During a busy slot, there is probability $p_{1|B} = 2/5$ (respectively, $p_{2|B} = 3/5$) that a task from user 1 (respectively, 2) is executed. We assume that events related to different slots are independent.
 - (a) Find the probability that a task from user 1 is executed for the first time during the 4th slot.
 - (b) Given that exactly 5 out of the first 10 slots were idle, find the probability that the 6th idle slot is slot 12.
 - (c) Find the expected number of slots up to and including the 5th task from user 1.
 - (d) Find the expected number of busy slots up to and including the 5th task from user 1.
 - (e) Find the PMF, mean, and variance of the number of tasks from user 2 until the time of the 5th task from user 1.

2. (Problem 5.11) Transmitters A and B independently send messages to a single receiver in a Poisson manner with average message arrival rates of λ_A and λ_B , respectively. All messages are so brief that we may safely assume that they occupy only single points in time.

The number of words in every message, regardless of its transmitting source, may be considered to be an independent experimental value of random variable W with PMF

$$p_W(w) = \begin{cases} 2/6 & , w = 1 \\ 3/6 & , w = 2 \\ 1/6 & , w = 3 \\ 0 & , \text{otherwise} \end{cases}$$

- (a) What is the probability that, during an interval of duration t , a total of exactly nine messages will be received?
- (b) Let N be the total number of words received during an interval of duration t . Determine the expected value for random variable N .
- (c) Determine the PDF for X , the time from $t = 0$ until the receiver has received exactly eight three-word messages from transmitter A.
- (d) Independent of what happens to all other words, a transmitter damages any particular word it sends with probability 10^{-3} . What is the probability that any particular damaged word is part of a three-word message?
- (e) What is the probability that exactly eight of the next twelve messages received will be from transmitter A?