Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

6.041/6.431: Probabilistic Systems Analysis

(Spring 2006)

Tutorial 08

April 13-14, 2006

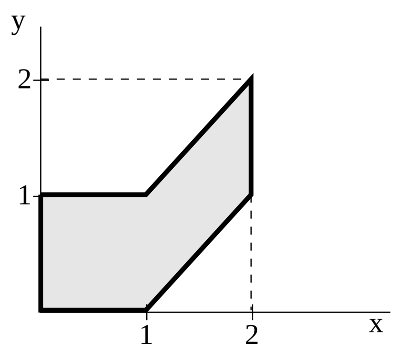
1. Suppose X is a unit normal random variable. Define a new random variable Y such that:

$$Y = a + bX + cX^2.$$

   Find the correlation coefficient ρ for X, Y.

2. Continuous random variables X and Y have a joint PDF given by

$$f_{X,Y}(x,y) = \begin{cases} C & \text{if } (x,y) \text{ belongs to the closed shaded region} \\ 0 & \text{otherwise} \end{cases}$$



   (a) The experimental value of X will be revealed to us; we have to design an estimator $g(X)$ of Y that minimizes the conditional expectation $E[(Y - g(X))^2 | X = x]$, for all x, over all possible estimators. Provide a plot of the optimal estimator as a function of its argument.

   (b) Let $g^*(X)$ be the optimal estimator of part (a). Find the numerical value of $E[g^*(X)]$ and $\text{var}(g^*(X))$?

   (c) Find the mean square error $E[(Y - g^*(X))^2]$. Is that the same as $E[\text{var}(Y \mid X)]$?

   (d) Find $\text{var}(Y)$.

3. Random variable X is uniformly distributed between -1.0 and 1.0. Let $X_1, X_2, \ldots,$ be independent identically distributed random variables with the same distribution as X. Determine which, if any, of the following sequences (all with $i = 1, 2, \ldots$) are convergent in probability. Give reasons for your answers. Include the limits if they exist.

   (a) X_i

   (b) $Y_i = \dfrac{X_i}{i}$

   (c) $Z_i = (X_i)^i$

   (d) $T_i = X_1 + X_2 + \ldots + X_i$

   (e) $U_i = \dfrac{X_1 + X_2 + \ldots + X_i}{i}$

   (f) $V_i = X_1 \cdot X_2 \cdot \ldots \cdot X_i$

   (g) $W_i = \max(X_1, \ldots, X_i)$