Tutorial 05 Answer
March 16-17, 2006

1. (a)

$$f_Y(y) = 2y, \text{for } 0 \le y \le 1.$$

(b)

$$f_Y(y) = e^{-y}, \text{ for } 0 \le y < \infty.$$

2. (a)

$$f_Y(y) = \frac{1}{\pi\sqrt{1-y^2}}, \text{for } -1 < y < 1.$$

(b)

$$f_Y(y) = \frac{1}{\pi} \cdot \frac{1}{1+y^2}, \text{ for } -\infty < y < \infty.$$

A random variable with the above density is called a Cauchy random variable

3. **Optional**

(a)

$$\begin{aligned} f_Y(y) &= \frac{1}{2\sqrt{y}} \cdot f_X(\sqrt{y}) + \frac{1}{2\sqrt{y}} \cdot f_X(-\sqrt{y}) \\ &= \frac{1}{\sqrt{y}} \cdot f_X(\sqrt{y}), \text{for } 0 \le y < \infty. \end{aligned}$$

(b)

$$f_Y(y) = \frac{1}{y} f_X(\ln y), \text{for } 0 \le y < \infty.$$

Note that f_X is the standard normal density in both (a) and (b).

4. (a)

$$f_{V,W}(v,w) = \frac{\log(1/v)}{2\sqrt{w}}, \quad 0 \le v, w \le 1$$

(b)

$$\begin{aligned} P(XY \le Z^2) &= P(V \le W) = \int_0^1 \int_0^w \frac{\log(1/v)}{2\sqrt{w}} dv dw = \int_0^1 \frac{v(1-\log v)}{2\sqrt{w}} |_{v=0}^{w} dw \\ &= \int_0^1 \frac{\sqrt{w}(1-\log w)}{2} dw = [\frac{w^{3/2}}{3}(\frac{5}{3} - \log w)]_{w=0}^1 = \frac{5}{9} \end{aligned}$$