

**Tutorial 05**  
**March 16-17, 2006**

1. For  $X$  a random variable uniformly distributed between -1 and 1, find the density function of  $Y$  where:
  - a)  $Y = \sqrt{|X|}$ .
  - b)  $Y = -\ln |X|$ .
2. a) Suppose a random variable  $X$  is uniformly distributed between 0 and 1. If  $Y = \cos \pi X$  find the density function for  $Y$ .  
  
b) Now suppose  $X$  is uniformly distributed between  $-1/2$  and  $1/2$ . Find the density function for  $Y$ , where  $Y = \tan \pi X$ .
3. **Optional** Suppose  $X$  is a standard normal random variable, i.e.  $X \sim N[0, 1]$ . Find the density for  $Y$ , where:
  - a)  $Y = X^2$ .
  - b)  $Y = e^X$ .
4. Let continuous random variables  $X$ ,  $Y$  and  $Z$  be independent and identically distributed according to the uniform distribution in the unit interval  $[0, 1]$ .
  - (a) Consider two new random variables defined by  $V = XY$  and  $W = Z^2$ . Derive the joint PDF  $f_{V,W}(v, w)$ .
  - (b) Show that  $\mathbf{P}(XY < Z^2) = \frac{5}{9}$ .