Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Tutorial
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1.

$$\begin{aligned} E[X] &= \boxed{\frac{1}{\lambda}(2p-1)} \\ Var(X) &= \boxed{\frac{2}{\lambda^2} - \frac{1}{\lambda^2}(2p-1)^2} \end{aligned}$$

2.

$$\mathbf{P}(1 \le X \le 3) = \frac{2}{3}\left(2\Phi\left(1\right) - 1\right) + \frac{1}{3}\left(2\Phi\left(\frac{1}{3}\right) - 1\right).$$

3.

(a) Using the total expectation theorem, we obtain
$\mathbf{E}[X] = \mathbf{E}[X|A]\mathbf{P}(A) + \mathbf{E}[X|B]\mathbf{P}(B) = 1 * \frac{1}{2} + \frac{1}{3} * \frac{1}{2} = \frac{2}{3}$

(b) Using the total probability theorem, we obtain
$\mathbf{P}(D) = \mathbf{P}(D|A)\mathbf{P}(A) + \mathbf{P}(D|B)\mathbf{P}(B) = \frac{1}{2}e^{-\tau} + \frac{1}{2}e^{-3\tau}$

(c) Using the Bayes' theorem, we obtain
$\mathbf{P}(T_{1A}|D) = \frac{1}{1 + e^{-2\tau}}$

(d) Using the total expectation theorem, we obtain
$\mathbf{E}[\text{Total Time Till Failure} \mid D]$
$= \tau + \mathbf{E}[\text{Time to failure after } \tau \; |D, A]\mathbf{P}(A|D) + \mathbf{E}[\text{Time to failure after } \tau \; |D, B]\mathbf{P}(B|D)$
$= \tau + \frac{1}{1+e^{-2\tau}} + (\frac{1}{3})\frac{e^{-2\tau}}{1+e^{-2\tau}}$