

Recitation 21

Markov Chains: Absorption Probabilities and Expected Time to Absorption
May 11, 2006

1. Josephina is currently a 6-1 student. On each day that she is a 6-1 student, she has a probability of $1/2$ of being a course 6-1 student the next day. Otherwise, she has an equally likely chance of becoming a 6-2 student, a 6-3 student, a course 9 student or a course 15 student the next day. On any day she is a 6-3 student, she has a probability of $1/4$ of switching to course 9, a probability of $3/8$ of switching to 6-1 and a probability of $3/8$ of switching to 6-2 the next day. On any day she is a 6-2 student, she has a probability of $1/2$ of switching to course 15, a probability of $3/8$ of switching to 6-1 and a probability of $1/8$ of switching to 6-3 the next day.

In answering the questions below, assume Josephina will be a student forever. Also assume, for parts (a)-(f) that if Josephina switches to course 9 or course 15, she will stay there and will not change her course again.

- (a) What is the probability that she eventually will leave course 6?
- (b) What is the probability that she will eventually be in course 15?
- (c) What is the expected number of days until she leaves course 6?
- (d) Every time she switches into 6-1 from 6-2 or 6-3, she buys herself an ice cream cone at Tosci's. She can only afford so much ice cream, so after she's eaten 2 ice cream cones, she stops buying herself ice cream. What is the expected number of ice cream cones she buys herself before she leaves course 6?
- (e) Her friend Oscar started out just like Josephina. He is now in course 15. You don't know how long it took him to switch. What is the expected number of days it took him to switch to course 15? [Hint: He had no particular aversion to course 9.]
- (f) Josephina decides that course 15 is not in her future. Accordingly, when she is a course 6-1 student, she stays 6-1 for another day with probability $1/2$, and otherwise she has an equally likely chance of becoming any of the other options. When she is 6-2, her probability of entering 6-1 or 6-3 are in the same proportion as before. What is the expected number of days until she is in course 9?
- (g) Suppose that if she is course 9 or course 15, she has probability $1/8$ of returning to 6-1, and otherwise she remains in her current course. What is the expected number of days until she is 6-1 again? (Notice that we know today she is 6-1, so if tomorrow she is still 6-1, then the number of days until she is 6-1 again is 1).