Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Recitation 14 Solutions
April 11, 2006

1. We know that:

$$\rho(X_1, X_2) = \frac{\text{Cov}(X_1, X_2)}{\sigma_{X_1}\sigma_{X_2}}$$

Therefore we first find the covariance:

$$\begin{aligned}\text{Cov}(A, B) &= \mathbf{E}[AB] - \mathbf{E}[A]\mathbf{E}[B] \\ &= \mathbf{E}[WX + WY + X^2 + XY] \\ &= \mathbf{E}[X^2] = 1\end{aligned}$$

and

$$\begin{aligned}\sigma_A &= \sqrt{\text{Var}(A)} = \sqrt{2} \\ \sigma_B &= \sqrt{\text{Var}(B)} = \sqrt{2}\end{aligned}$$

and therefore:

$$\rho(A, B) = \frac{1}{2}.$$

We proceed as above to find the correlation of A, C.

$$\begin{aligned}\text{Cov}(A, C) &= \mathbf{E}[AC] - \mathbf{E}[A]\mathbf{E}[C] \\ &= \mathbf{E}[WY + WZ + XY + XZ] \\ &= 0\end{aligned}$$

and therefore

$$\rho(A, C) = 0.$$

2. Solution is in the text, pp. 264–265.

3. Solution is in the text, pp. 267–268.